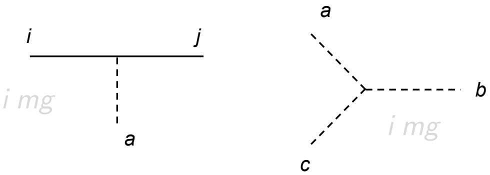
Pomeron in a toy model

Energy behavior $\sigma \propto s^\delta$ with non-integer δ looks a bit strange. How can we derive it from Feynman diagrams?

Consider a theory with massless scalar "quarks" and massive scalar "gluons" (m), which interact via



Consider small-angle elastic qq scattering. We want to calculate

$$\mathrm{Im}\mathcal{M}(s,t) \propto s^{\alpha_{\mathbf{P}}(t)}\,,$$

and derive the Pomeron intercept $\alpha_{\mathbf{P}}(t)$ in this model.

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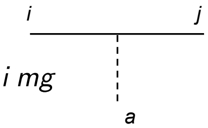

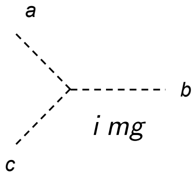

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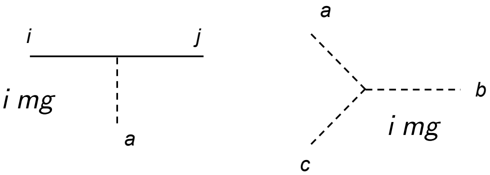

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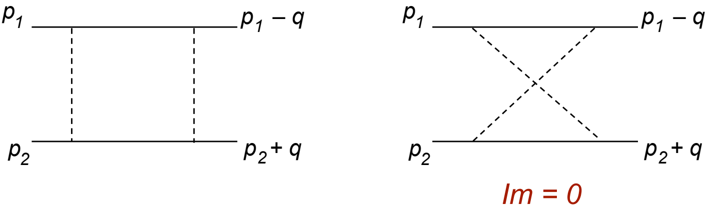
Pomeron in a toy model: 1-loop level



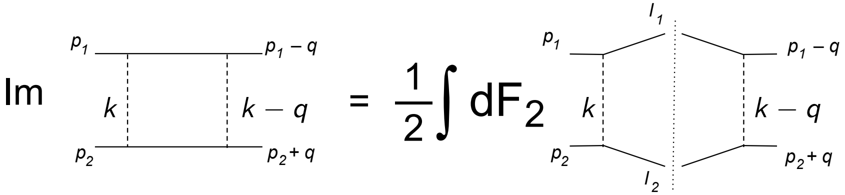
Lowest order (1 loop): exchange of two "gluons".

In real QCD we must start with two gluons in a color-singlet state because the Pomeron does not carry any color.

Here we just note that the imaginary part starts from one loop.

Pomeron in a toy model: 1-loop level

Applying Cutkosky's rules (generalization of the optical theorem):



$$\mathrm{Im}\mathcal{M}^{(1)} = \frac{1}{2}\int dF_2\,\mathcal{A}_{2\to 2}(k)\cdot\mathcal{A}^{\dagger}_{2\to 2}(k-q)\,,$$

where

$$dF_2 = \frac{d^4\ell_1}{(2\pi)^3}\delta(\ell_1^2)\,\frac{d^4\ell_2}{(2\pi)^3}\delta(\ell_2^2)\cdot(2\pi)^4\delta^{(4)}(p_1+p_2-\ell_1-\ell_2)\,.$$

$$\mathcal{A}_{2\to 2}(k) = \frac{m^2g^2}{k^2-m^2+i\epsilon}\,.$$

Pomeron in a toy model: 1-loop level

Using $\ell_1^\mu = p_1^\mu - k^\mu$, $\ell_2^\mu = p_2^\mu + k^\mu$, $d^4\ell_1 \to d^4k$, we get:

$$\mathrm{Im}\mathcal{M}^{(1)} = \frac{1}{8\pi^2}\int d^4k\,\delta[(p_1-k)^2]\delta[(p_2+k)^2]\,\frac{m^4g^4}{(k^2-m^2)((k-q)^2-m^2)}\,.$$

It is now convenient to use Sudakov variables. In the center of motion frame:

$$p_1^\mu = \frac{\sqrt{s}}{2}(1,0,0,1)\,,\quad p_2^\mu = \frac{\sqrt{s}}{2}(1,0,0,-1)\,,$$

so that $p_1^2 = 0 = p_2^2$, $(p_1+p_2)^2 = 2p_1p_2 = s$. Then "gluon" momentum is

$$k^\mu = \alpha p_1^\mu - \beta p_2^\mu + k_\perp^\mu\,,\quad d^4k = \frac{s}{2}d\alpha d\beta d^2\mathbf{k}\,.$$

where $\mathbf{k}$ is the 2D transverse momentum.

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Pomeron in a toy model: 1-loop level

2 delta-functions:

$$(p_1 - k)^2 = \beta(1-\alpha)s - \mathbf{k}^2 = 0\,,$$
$$(p_2 + k)^2 = \alpha(1-\beta)s - \mathbf{k}^2 = 0\,.$$

Important observation: typical $\mathbf{k}^2 \ll s$, so $\alpha = \beta \approx \mathbf{k}^2/s \ll 1$. Therefore,

$$\int d\alpha d\beta\, \delta[(p_1-k)^2]\delta[(p_2+k)^2] \approx \int d\alpha d\beta\, \delta(\beta s - \mathbf{k}^2)\delta(\alpha s - \mathbf{k}^2) = \frac{1}{s^2}\,.$$

Also, $k^2 = \alpha\beta s - \mathbf{k}^2 \approx -\mathbf{k}^2$ and $(k-q)^2 \approx -(\mathbf{k}-\mathbf{q})^2$. 1-loop result:

$$\mathrm{Im}\mathcal{M}^{(1)} = \frac{m^4 g^4}{16\pi^2 s}\int \frac{d^2\mathbf{k}}{(\mathbf{k}^2+m^2)((\mathbf{k}-\mathbf{q})^2+m^2)}\,.$$

At this level, $\alpha_{\mathbb{P}} = -1$.

Pomeron in a toy model: 1-loop level

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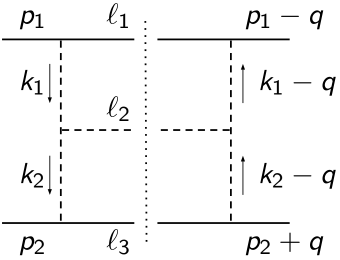
Pomeron in a toy model: 2-loop level

Consider the following ladder diagram:

$$\mathrm{Im}\mathcal{M}^{(2)} = \frac{1}{2}\int dF_3\, \mathcal{A}_{2\to 3}(k_1, k_2) \times \mathcal{A}^{\dagger}_{2\to 3}(k_1 - q, k_2 - q)\,,$$



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and

$$\mathcal{A}_{2\to 3}(k_1, k_2) = \frac{m^6 g^6}{(k_1^2 - m^2)(k_2^2 - m^2)}\,.$$

Pomeron in a toy model: 2-loop level

Longitudinal integral:

$$\int d\alpha_1\, d\beta_1\, d\alpha_2\, d\beta_2\, \delta(\ell_1^2)\delta(\ell_2^2)\delta(\ell_3^2)\,.$$

Three integrals are killed, one remains. Using Sudakov variables, $k_i = \alpha_i p_1 - \beta_i p_2 + k_{i\perp}$, we get:

$$\begin{aligned}
\ell_1^2 &= (1-\alpha_1)\beta_1 s - \mathbf{k}_1^2 = 0\,,\\
\ell_2^2 &= (\alpha_1-\alpha_2)(\beta_2-\beta_1)s - (\mathbf{k}_1-\mathbf{k}_2)^2 = 0\,,\\
\ell_3^2 &= (1-\beta_2)\alpha_2 s - \mathbf{k}_2^2 = 0\,.
\end{aligned}$$

There is a logarithmically enhanced contribution from the region

$$1 \gg \alpha_1 \gg \alpha_2 \approx \frac{\mathbf{k}_2^2}{s}\,, \quad \frac{\mathbf{k}_1^2}{s} \approx \beta_1 \ll \beta_2 \ll 1\,, \quad \alpha_1\beta_2 = \frac{(\mathbf{k}_1-\mathbf{k}_2)^2}{s}\,.$$

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Pomeron in a toy model: 2-loop level

The longitudinal integral then becomes

$$\frac{1}{s^2}\int d\alpha_1\, d\beta_2\, \delta(\alpha_1\beta_2 s - (\mathbf{k}_1 - \mathbf{k}_2)^2) =$$
$$= \frac{1}{s^3}\int \frac{d\alpha_1}{\alpha_1} = \frac{1}{s^3}\log\left(\frac{\alpha_{1max}}{\alpha_{1min}}\right) \approx \frac{1}{s^3}\log\left(\frac{s}{|t|}\right).$$

Note that the exact limits α_{1max} and α_{1min} are inessential when we only care about logarithmic contribution.

$$\mathrm{Im}\mathcal{M}^{(2)} = \frac{m^6 g^6}{256\pi^5 s}\log\left(\frac{s}{|t|}\right)\left[\int \frac{d^2\mathbf{k}}{(\mathbf{k}^2+m^2)((\mathbf{k}-\mathbf{q})^2+m^2)}\right]^2$$
$$= \mathrm{Im}\mathcal{M}^{(1)}\cdot x\,, \quad \text{where} \quad x = \frac{m^2 g^2}{16\pi^3}\log\left(\frac{s}{|t|}\right)\int \frac{d^2\mathbf{k}}{(\mathbf{k}^2+m^2)((\mathbf{k}-\mathbf{q})^2+m^2)}$$

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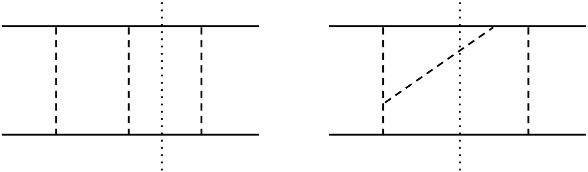
Pomeron in a toy model: 2-loop level

No other 2-loop diagram is not log s-enhanced!



These diagrams which do not contribute to the leading logarithmic approximation in the scalar toy model (Problem 2: prove this!), but they become important in QCD.

Pomeron in a toy model: n-loop level

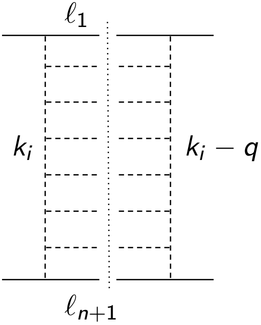
At the n-loop level, $\mathcal{M} \propto (g^2)^{n+1}$. The largest power of logarithm is $\log^n(s/|t|)$, and it comes from the $(n-1)$-rung ladder diagram.

As usual, $k_i^\mu = \alpha_i p_1^\mu - \beta_i p_2^\mu + k_{\perp i}^\mu$, $i = 1, \ldots, n$.
The $\log^n$ enhancement comes from

$$\int d\alpha_1 d\beta_1 \cdots d\alpha_n d\beta_n \, \delta(\ell_1^2) \cdots \delta(\ell_{n+1}^2)$$

in the multi-Regge kinematical region

$$1 \gg \alpha_1 \gg \cdots \gg \alpha_{n-1} \gg \alpha_n \approx |t|/s \,,$$
$$|t|/s \approx \beta_1 \ll \beta_2 \ll \cdots \ll \beta_n \ll 1 \,.$$



Pomeron in a toy model: n-loop level

The longitudinal integral becomes

$$\int_{|t|/s}^{1} \frac{d\alpha_{n-1}}{\alpha_{n-1}} \int_{\alpha_{n-1}}^{1} \frac{d\alpha_{n-2}}{\alpha_{n-2}} \cdots \int_{\alpha_3}^{1} \frac{d\alpha_2}{\alpha_2} \int_{\alpha_2}^{1} \frac{d\alpha_1}{\alpha_1} = \frac{1}{n!} \log\left(\frac{s}{|t|}\right)^n .$$

The transverse integrals are all of the same type as before. Therefore, we get for the n-loop contribution to the leading logarithmic approximation: $\mathrm{Im}\mathcal{M}^{(n)} = \mathrm{Im}\mathcal{M}^{(1)} \cdot x^n/n!$ with the same x as before. Resumming all loops in the leading logarithmic approximation gives:

$$\mathrm{Im}\mathcal{M} = \mathrm{Im}\mathcal{M}^{(1)} \left(1 + x + \frac{x^2}{2!} + \cdots \right) = \mathrm{Im}\mathcal{M}^{(1)} \cdot e^x .$$

Pomeron in a toy model

Substituting x, we get

$$\mathrm{Im}\mathcal{M} \propto s^{\alpha_{\mathbb{P}}(t)}\,, \text{ where } \alpha_{\mathbb{P}} = -1 + \frac{g^2 m^2}{16\pi^3}\int \frac{d^2\mathbf{k}}{(\mathbf{k}^2 + m^2)((\mathbf{k}-\mathbf{q})^2 + m^2)}\,.$$

In particular,

$$\alpha_{\mathbb{P}}(0) = -1 + \frac{g^2}{16\pi^2}\,.$$

This is how non-integer power of the energy appears in diagrammatic calculations.

Pomeron in perturbative QCD

Pomeron in the true perturbative QCD (BFKL-Pomeron) was rigorously derived in works by *Balitski, Fadin, Kuraev, Lipatov (1976–1978).* It bears several additional complications: numerators are non-trivial, extra diagrams must be included, and t-channel gluons are reggeized, the singularity is a cut rather than a pole, etc.

The result for the leading singularity ($\approx$ the Pomeron intercept) in the leading log approximation is:

$$\alpha_{\mathbb{P}}(0) = 1 + \frac{4N\alpha_s}{\pi} \log 2\,,$$

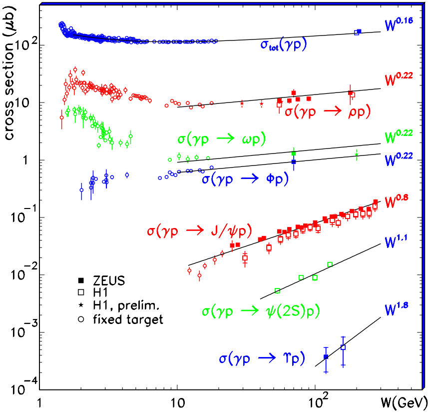
where $N = 3$ is the number of colors.

Pomeron in perturbative QCD

The result looks nice but it brings several uneasy questions.

The intercept of the BFKL Pomeron is large: at $\alpha_s = 0.15$, we get $\alpha_{\mathbb{P}} \approx 1.4$, so that $\sigma \propto s^{2(\alpha_{\mathbb{P}}-1)} = s^{0.8}$, which is a way too steep!

It also shows a wrong dependence on hardness of the scattering: compare with diffractive vector meson production $\gamma p \to Vp$, $V = \rho, \omega, \phi, J/\psi, \Upsilon$.



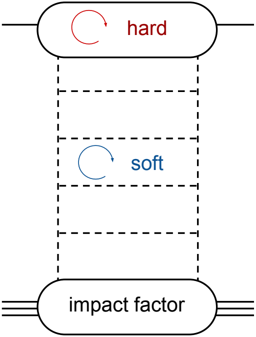
Pomeron in perturbative QCD

This means that the BFKL Pomeron derived from perturbative QCD does not exhaust all the physics of the Pomeron exchange.

There is an additional contribution ("soft Pomeron") with very small intercept which dominates the scattering in the soft region.

Other problems with applying the BFKL Pomeron to data:

- Pomeron couples to particles via impact factors: simple for q, g, γ, but are impossible to calculate exactly for hadrons $\to$ must use some phenomenological input.
- Soft-to-hard diffusion: even if the first and last loops are hard ($\mathbf{k}$ are large), the middle loops can be soft. Validity of the perturbative approach is then doubtful.



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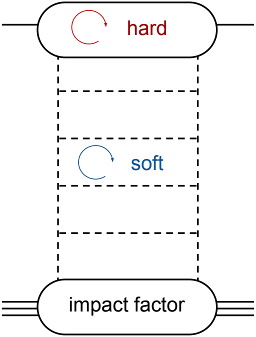

Pomeron in perturbative QCD

Formally, the BFKL Pomeron violates Froissart bound. Therefore, at high energy (or high partonic density) one must include multi-Pomeron exchanges. The Pomerons can interact with each other $\to$ a very complicated process, which is not yet understood!

Remains a hot topic of research, which is of much importance for the LHC. Keywords: *unitarization of the Pomeron, saturation of partonic densities, non-linear evolution equations*, etc.

Central exclusive production

One particularly interesting process at the LHC is Higgs central exclusive production (CEP). Very roughly, this is a "$I\!P I\!P$ collision" leading to creation of the Higgs boson. Each proton "emits" a Pomeron losing a small fraction of its energy, and the produced Higgs stays at midrapidity.

Although this process is QCD-driven, it has extremely clean experimental signature: the central detector sees only the H decay products (no soft hadrons, no proton remnants!), while forward detectors tag the two protons. For some time it was even suggested as the Higgs discovery channel!

It can detect the Higgs even if it decays completely invisibly! Although we don't see the decay products, we do see the tagged protons and we can find their energy loss $\to$ invariant mass distribution will have a peak.

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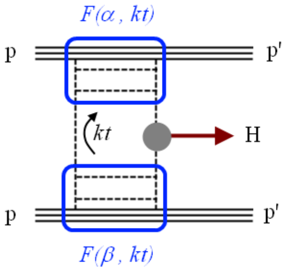
Higgs CEP

This could be a golden channel for the Higgs boson production in the mass region $M_H = 115 - 130$ GeV, where it decays mostly into $b\bar{b}$.

Forward scattering; $\alpha, \beta \ll 1$ are momentum fractions lost by protons.



$$d\sigma \propto \left| \int \frac{d^2 k_\perp}{k_\perp^4} \mathcal{F}_g(\alpha, k_\perp^2) \mathcal{F}_g(\beta, k_\perp^2) \cdot S(m_H^2, k_\perp^2) \cdot M(gg \to H) \right|^2 \cdot \text{GSP}\,.$$

Here $\mathcal{F}_g$ are the unintegrated gluon distributions, $S(m_H^2, k_\perp^2)$ is the Sudakov formfactor; GSP is the "gap survival probability".

Some of these factors are closely related to the Pomeron properties.

Higgs CEP

$$d\sigma \propto \left| \int \frac{d^2 k_\perp}{k_\perp^4} \mathcal{F}_g(\alpha, k_\perp^2) \mathcal{F}_g(\beta, k_\perp^2) \cdot S(m_H^2, k_\perp^2) \cdot M(gg \to H) \right|^2 \cdot \mathrm{GSP}\,.$$

- the unintegrated gluon distributions $\mathcal{F}_g$ is a phenomenological way to take into account the impact factor of the proton (not calculable in pQCD) and the Pomeron itself. Parametrizatoins exist, but they are quite nucertain for small $k_\perp < 1$ GeV.
- The Sudakov formfactor $S(m_H^2, k_\perp^2)$ describes absence of partonic emission during gg fusion in the colorless Higgs. It is quite uncertain and is a matter of debates.
- The gap survival probability GSP is related to the protons rescattering. It is also not known very well and depends on the Pomeron properties.

Higgs CEP

- The first attempts to calculate the Higgs CEP in 1990's gave results different by three order of magnitude! The main origin of discrepancy was neglecting some of these factors.
- Now, predictions for the LHC are $\sigma \sim \mathcal{O}(1\,\mathrm{fb})$, but remain quite uncertain, an order of magnitude up or down. These uncertainties can be reduced if CEP of other states (jj, $\gamma\gamma$, $\ell^+\ell^-$) is measured.
- Still, central exclusive diffraction (not necessarily Higgs) is definitely worth studying at the LHC because of very clean signature.