Abelian Higgs model

$m^2 \to -\mu^2$

$$L = (D_\mu \varphi)^* (D^\mu \varphi) + \mu^2 \varphi^* \varphi - \frac{\lambda}{4} (\varphi^* \varphi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\varphi = \frac{1}{\sqrt{2}} (v + h) e^{i\theta/v} \qquad v^2 = \frac{4\mu^2}{\lambda}$$

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$$\varphi = \frac{1}{\sqrt{2}}(v+h)e^{i\theta/v} \qquad v^2 = \frac{4\mu^2}{\lambda}$$

Unitary gauge $\theta = 0$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} \left[\partial_\mu h - ie(v+h)A_\mu\right]$$

$$L = \frac{1}{2} (\partial_\mu h)(\partial^\mu h) + \frac{e^2}{2}(v+h)^2 A_\mu A^\mu - \mu^2 h^2 - \frac{\lambda v}{4} h^3 - \frac{\lambda}{16} h^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Photon

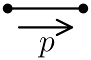
$$L_A = \frac{1}{2} A^\mu M_{\mu\nu} A^\nu$$

$$M_{\mu\nu}(\partial) = g_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu + m_A^2 g_{\mu\nu} \qquad m_A = ev$$

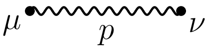
$$M_{\mu\nu}(-ip) = -\left(p^2 - m_A^2\right)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + m_A^2 \frac{p_\mu p_\nu}{p^2}$$

$$M_{\mu\nu}^{-1}(-ip) = -\frac{1}{p^2 - m_A^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \frac{1}{m_A^2}\frac{p_\mu p_\nu}{p^2}$$

Unitary gauge: propagators

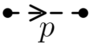


$$= iG(p) \qquad G(p) = \frac{1}{p^2 - m_h^2 + i0}$$



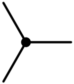
$$= -iD_{\mu\nu}(p)$$

$$D_{\mu\nu}(p) = \frac{1}{p^2 - m_A^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}\right]$$

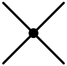


$$= iG_c(p) \qquad G_c(p) = -\frac{1}{m_c^2}$$

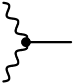
Unitary gauge: vertices



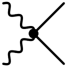
$$= -\frac{3}{2} i\lambda v$$



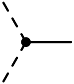
$$= -\frac{3}{2} i\lambda$$



$$= 2ie^2 v g_{\mu\nu}$$



$$= 2ie^2 g_{\mu\nu}$$



$$= -i\frac{m_c^2}{v}$$

Renormalizable gauge

$$\varphi = \frac{1}{\sqrt{2}}(v + h + ib)$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}}\left[\partial_\mu h + ebA_\mu + i\partial_\mu b - ie(v+h)A_\mu\right]$$

$$V(\varphi) = \mu^2 h^2 + \frac{\lambda v}{2} h(h^2 + b^2) + \frac{\lambda}{16}(h^2 + b^2)^2$$

$$L = \frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) - \mu^2 h^2 + \frac{1}{2}\left(\partial_\mu b\right)\left(\partial^\mu b\right) - evA^\mu \partial_\mu b$$

$$- \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{e^2 v^2}{2} A_\mu A^\mu$$

$$- \frac{\lambda v}{2} h(h^2 + b^2) - \frac{\lambda}{16}(h^2 + b^2)^2 + eA^\mu\left(b\partial_\mu h - h\partial_\mu b\right)$$

$$+ e^2 v h A_\mu A^\mu + \frac{e^2}{2}(h^2 + b^2) A_\mu A^\mu$$

R_ξ gauge

Gauge $G = 0$

$$L_\xi = -\frac{1}{2\xi}G^2 \qquad L_c \sim \bar{c}\frac{\delta G}{\delta \alpha}c$$

R_ξ gauge

Gauge $G = 0$

$$L_\xi = -\frac{1}{2\xi} G^2 \qquad L_c \sim \bar{c} \frac{\delta G}{\delta \alpha} c$$

$$G = \partial_\mu A^\mu + \xi e v b$$

$$L_\xi = -\frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2 - e v b \partial_\mu A^\mu - \frac{\xi e^2 v^2}{2} b^2$$

R_ξ gauge

Gauge $G = 0$

$$L_\xi = -\frac{1}{2\xi} G^2 \qquad L_c \sim \bar{c} \frac{\delta G}{\delta \alpha} c$$

$$G = \partial_\mu A^\mu + \xi e v b$$

$$L_\xi = -\frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2 - e v b \partial_\mu A^\mu - \frac{\xi e^2 v^2}{2} b^2$$

$$\delta A_\mu = \frac{1}{e} \partial_\mu \delta\alpha \qquad \delta\varphi = i \delta\alpha \varphi \qquad \delta b = (v + h) \delta\alpha$$

$$\delta G = \left[\frac{1}{e} \partial^2 + \xi e v (v + h)\right] \delta\alpha$$

$$L_c = \left(\partial_\mu \bar{c}\right)\left(\partial^\mu c\right) - \xi e^2 v (v + h) \bar{c} c$$

Full Lagrangian

$$L + L_\xi + L_c = \frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) - \frac{m_h^2}{2}h^2$$
$$+ \frac{1}{2}\left(\partial_\mu b\right)\left(\partial^\mu b\right) - \frac{\xi m_A^2}{2}b^2$$
$$+ \left(\partial_\mu \bar{c}\right)\left(\partial^\mu c\right) - \xi m_A^2 \bar{c}c$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + \frac{m_A^2}{2}A_\mu A^\mu$$
$$- \frac{\lambda v}{2}h(h^2 + b^2) - \frac{\lambda}{16}(h^2 + b^2)^2$$
$$+ eA^\mu\left(b\partial_\mu h - h\partial_\mu b\right) + e^2 v h A_\mu A^\mu + \frac{e^2}{2}(h^2 + b^2)A_\mu A^\mu$$
$$- \xi e^2 v h \bar{c}c$$

$$m_h = \sqrt{2}\mu \qquad m_A = ev \qquad v = \frac{2\mu}{\sqrt{\lambda}}$$

Photon

$$L = \frac{1}{2} A^\mu M_{\mu\nu} A^\nu$$

$$M_{\mu\nu}(\partial) = g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{\xi}\partial_\mu\partial_\nu + m_A^2 g_{\mu\nu}$$

$$M_{\mu\nu}(-ip) = -\left(p^2 - m_A^2\right)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \left(\frac{p^2}{\xi} - m_A^2\right)\frac{p_\mu p_\nu}{p^2}$$

$$M_{\mu\nu}^{-1}(-ip) = -\frac{1}{p^2 - m_A^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{\xi}{p^2 - \xi m_A^2}\frac{p_\mu p_\nu}{p^2}$$

$$D_{\mu\nu}(p) = \frac{1}{p^2 - m_A^2}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_A^2}\right]$$

Particular cases

'**t Hooft–Landau gauge** $\xi \to 0$

$$G_b(p) = G_c(p) = \frac{1}{p^2} \qquad D_{\mu\nu}(p) = \frac{1}{p^2 - m_A^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right]$$

Particular cases

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't Hooft–Feynman gauge $\xi = 1$

$$G_b(p) = G_c(p) = \frac{1}{p^2 - m_A^2} \qquad D_{\mu\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_A^2}$$

Particular cases

't Hooft–Landau gauge $\xi \to 0$

$$G_b(p) = G_c(p) = \frac{1}{p^2} \qquad D_{\mu\nu}(p) = \frac{1}{p^2 - m_A^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right]$$

't Hooft–Feynman gauge $\xi = 1$

$$G_b(p) = G_c(p) = \frac{1}{p^2 - m_A^2} \qquad D_{\mu\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_A^2}$$

Unitary gauge $\xi \to \infty$

$$D_{\mu\nu}(p) = \frac{1}{p^2 - m_A^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}\right)$$

b disappears; $c \to c/\sqrt{\xi}$

$$L_c = -m_A^2 \bar{c}c - e^2 v h \bar{c}c$$

Nonabelian gauge fields

$$L = (\partial_\mu \varphi)^+ (\partial^\mu \varphi)$$

Invariant with respect to $SU(N)$: $\varphi(x) \to U\varphi(x)$

Infinitesimal tramsformation

$$U = 1 + i\alpha^a T^a$$

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How to make it invariant for $\varphi(x) \to U(x)\varphi(x)$?

$$\partial_\mu \varphi \;\Rightarrow\; D_\mu \varphi$$

$$D_\mu \varphi = (\partial_\mu - igA_\mu)\varphi \qquad A_\mu = A_\mu^a T^a$$

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$$D_\mu \varphi = (\partial_\mu - igA_\mu)\varphi \qquad A_\mu = A_\mu^a T^a$$

$$\varphi \to \varphi' = U\varphi \qquad A_\mu \to A'_\mu \qquad D_\mu \varphi \to D'_\mu \varphi' = U D_\mu \varphi$$
$$(\partial_\mu - igA'_\mu)U\varphi = U(\partial_\mu - igA_\mu)\varphi$$
$$\partial_\mu U - igA'_\mu U = -igUA_\mu$$
$$A'_\mu = UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}$$

Infinitesimal tramsformation

$$\varphi(x) \to \varphi'(x) = (1 + i\alpha^a(x)T^a)\varphi(x)$$

$$A^a_\mu(x) \to A'^a_\mu(x) = A^a_\mu(x) + \frac{1}{g}D^{ab}_\mu \alpha^b(x)$$

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - ig(T^c)^{ab}A^c_\mu$$

$$(T^c)^{ab} = if^{acb} \qquad [T^a, T^b] = if^{abc}T^c$$

Field strength

$$
\begin{aligned}
&[D_\mu, D_\nu]\varphi \\
&= \partial_\mu\partial_\nu\varphi - ig(\partial_\mu A_\nu)\varphi - igA_\nu\partial_\mu\varphi - igA_\mu\partial_\nu\varphi - g^2 A_\mu A_\nu\varphi \\
&- \partial_\nu\partial_\mu\varphi + ig(\partial_\nu A_\mu)\varphi + igA_\mu\partial_\nu\varphi + igA_\nu\partial_\mu\varphi + g^2 A_\nu A_\mu\varphi \\
&= -igG_{\mu\nu}\varphi \\
&G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] = G^a_{\mu\nu}T^a \\
&G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu
\end{aligned}
$$

$$
G_{\mu\nu} \to UG_{\mu\nu}U^{-1} \qquad G^a_{\mu\nu} \to U^{ab}G^b_{\mu\nu}
$$

Quantization

$$L = \left(D_\mu \varphi\right)^+ \left(D^\mu \varphi\right) - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

Gauge

$$G^a(A) = \frac{1}{\xi} \partial^\mu A^a_\mu$$

$$L_\xi = -\frac{1}{2\xi} \left(\partial^\mu A^a_\mu\right)^2$$

Quantization

$$L = \left(D_\mu \varphi\right)^+ \left(D^\mu \varphi\right) - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

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$$G^a(A) = \frac{1}{\xi} \partial^\mu A^a_\mu$$

$$L_\xi = -\frac{1}{2\xi} \left(\partial^\mu A^a_\mu\right)^2$$

$$\delta G^a = \frac{1}{g} \partial^\mu D^{ab}_\mu \alpha^b$$

$$L_c = \left(\partial_\mu \bar{c}\right) \left(D^\mu c\right)$$

Standard Model

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$L = (\partial_\mu \varphi)^+ (\partial^\mu \varphi) - V(\varphi)$$

$SU(2) \times U(1)$

$$\varphi \to U\varphi \approx (1 + i\alpha^a T^a)\varphi$$

$$\varphi \to e^{i\beta}\varphi \approx (1 + i\beta)\varphi$$

Standard Model

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$
$$L = (\partial_\mu \varphi)^+ (\partial^\mu \varphi) - V(\varphi)$$

$SU(2) \times U(1)$

$$\varphi \to U\varphi \approx (1 + i\alpha^a T^a)\varphi$$
$$\varphi \to e^{i\beta}\varphi \approx (1 + i\beta)\varphi$$

Make both symmetries local ($Y = \frac{1}{2}$)

$$D_\mu \varphi = \left(\partial_\mu - ig_2 A_\mu^a T^a - ig_1 Y B_\mu\right)\varphi$$

$$L = (D_\mu \varphi)^+ (D^\mu \varphi) - V(\varphi) - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_2 \varepsilon^{abc} A^b_\mu A^c_\nu \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

symmetric with respect to

$$\varphi \to (1 + i\alpha^a T^a + iY\beta)\varphi$$

$$A^a_\mu \to A^a_\mu + \frac{1}{g_2} D^{ab}_\mu \alpha^b \qquad D^{ab}_\mu = \delta^{ab}\partial_\mu + g_2 \varepsilon^{acb} A^c_\mu$$

$$B_\mu \to B_\mu + \frac{1}{g_1} \partial_\mu \beta$$

$$L = (D_\mu \varphi)^+ (D^\mu \varphi) - V(\varphi) - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_2 \varepsilon^{abc} A^b_\mu A^c_\nu \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

symmetric with respect to

$$\varphi \to (1 + i\alpha^a T^a + iY\beta)\varphi$$

$$A^a_\mu \to A^a_\mu + \frac{1}{g_2} D^{ab}_\mu \alpha^b \qquad D^{ab}_\mu = \delta^{ab}\partial_\mu + g_2 \varepsilon^{acb} A^c_\mu$$

$$B_\mu \to B_\mu + \frac{1}{g_1} \partial_\mu \beta$$

Vacuum expectation value

$$<\varphi> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

symmetric with respect to $U(1)$ $\alpha^1 = \alpha^2 = 0$, $\alpha^3 = \beta$

Unitary gauge

Make a gauge transformation

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

Gauge bosons

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right)$$
$$W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$$
$$A_{\mu\nu}^3 = \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3$$

Kinetic terms

$$L_k = -\frac{1}{2}W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4}A_{\mu\nu}^3 A^{3\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Gauge bosons

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}} \left(A^1_{\mu} \mp i A^2_{\mu} \right)$$
$$W^{\pm}_{\mu\nu} = \partial_{\mu} W^{\pm}_{\nu} - \partial_{\nu} W^{\pm}_{\mu}$$
$$A^3_{\mu\nu} = \partial_{\mu} A^3_{\nu} - \partial_{\nu} A^3_{\mu}$$

Kinetic terms

$$L_k = -\frac{1}{2} W^{+}_{\mu\nu} W^{-\mu\nu} - \frac{1}{4} A^3_{\mu\nu} A^{3\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$D_{\mu}\varphi = -i \left\{ \frac{g_2 v}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} \left(W^{+}_{\mu} T^{+} + W^{-}_{\mu} T^{-} \right) + A^3_{\mu} T^3 \right] + \frac{g_1 v}{2\sqrt{2}} B_{\mu} \right\} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \cdots$$

$$T^{\pm} = T^1 \pm i T^2 \qquad T^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad T^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$D_\mu \varphi = i\frac{v}{2}\left(\begin{array}{c} -g_2 W_\mu^+ \\ \frac{1}{\sqrt{2}}\left(g_2 A_\mu^3 - g_1 B_\mu\right) \end{array} \right) + \cdots$$

$$D_\mu \varphi = i\frac{v}{2}\begin{pmatrix} -g_2 W_\mu^+ \\ \frac{1}{\sqrt{2}}\left(g_2 A_\mu^3 - g_1 B_\mu\right) \end{pmatrix} + \cdots$$

Mass terms

$$L_m = \frac{g_2^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{\left(g_1^2 + g_2^2\right) v^2}{8}\left(A_\mu^3 \cos\theta_w - B_\mu \sin\theta_w\right)^2$$

where

$$\cos\theta_w = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \qquad \sin\theta_w = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$Z_\mu = A^3_\mu \cos\theta_w - B_\mu \sin\theta_w$$
$$A_\mu = A^3_\mu \sin\theta_w + B_\mu \cos\theta_w$$

$$\begin{aligned} L_0 = & -\frac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} + m_W^2 W^+_\mu W^{-\mu} \\ & - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{m_Z^2}{2} Z_\mu Z^\mu \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \end{aligned}$$

where

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and

$$m_W = \frac{g_2 v}{2} \qquad m_Z = \frac{\sqrt{g_1^2 + g_2^2}\, v}{2} \qquad m_W = m_Z \cos\theta_w$$

Higgs boson

$$L_h = \frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) - V(\varphi)$$

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^+\varphi - \frac{v^2}{2}\right)^2 = \frac{\lambda v^2}{4}h^2 + \frac{\lambda v}{4}h^3 + \frac{\lambda}{16}h^4$$

$$m_h^2 = \frac{\lambda v^2}{2}$$

Higgs – gauge bosons interactions

$$\frac{g_2^2(v+h)^2}{4}W_\mu^+W^{-\mu} = m_W^2\left(1+\frac{h}{v}\right)^2 W_\mu^+W^{-\mu}$$

interactions

$$2\frac{m_W^2}{v}hW_\mu^+W^{-\mu}+\frac{m_W^2}{v^2}h^2W_\mu^+W^{-\mu} = m_W g_2 hW_\mu^+W^{-\mu}+\frac{g_2^2}{4}h^2W_\mu^+W^{-}$$

Higgs – gauge bosons interactions

$$\frac{g_2^2 (v+h)^2}{4} W_\mu^+ W^{-\mu} = m_W^2 \left(1 + \frac{h}{v}\right)^2 W_\mu^+ W^{-\mu}$$

interactions

$$2\frac{m_W^2}{v} h W_\mu^+ W^{-\mu} + \frac{m_W^2}{v^2} h^2 W_\mu^+ W^{-\mu} = m_W g_2 h W_\mu^+ W^{-\mu} + \frac{g_2^2}{4} h^2 W_\mu^+ W^{-}$$

$$\frac{\left(g_1^2 + g_2^2\right)(v+h)^2}{8} Z_\mu Z^\mu = m_Z^2 \left(1 + \frac{h}{v}\right)^2 Z_\mu Z^\mu$$

interactions

$$2\frac{m_Z^2}{v} h Z_\mu Z^\mu + \frac{m_Z^2}{v^2} h^2 Z_\mu Z^\mu = m_Z \sqrt{g_1^2 + g_2^2}\, h Z_\mu Z^\mu + \frac{g_1^2 + g_2^2}{4} h^2 Z_\mu Z^\mu$$

Gauge-bosons interactions

$$
\begin{aligned}
-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} &= (\text{quadratic}) \\
&\quad - g_2\varepsilon^{abc}\left(\partial_\mu A^a_\nu\right)A^b_\mu A^c_\nu - \frac{g_2^2}{4}\varepsilon^{abc}\varepsilon^{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu \\
&= (\text{quadratic}) \\
&\quad - g_2\varepsilon^{abc}\left(\partial_\mu A^a_\nu\right)A^b_\mu A^c_\nu - \frac{g_2^2}{4}\left[\left(A^a_\mu A^{a\mu}\right)^2 - A^a_\mu A^a_\nu A^b_\mu A^b_\nu\right]
\end{aligned}
$$

3-boson vertices

$$L_3 = ig_2\Big[\frac{1}{2}\left(\partial_\mu A^3_\nu\right)\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right) + \left(\partial_\mu W^+_\nu\right)\left(W^-_\mu A^3_\nu - A^3_\mu W^-_\nu\right) + \left(\partial_\mu W^-_\nu\right)\left(A^3_\mu W^+_\nu - W^+_\mu A^3_\nu\right)\Big]$$

where

$$A^3_\mu = Z_\mu \cos\theta_w + A_\mu \sin\theta_w$$

4-boson vertices

$$L_4 = \frac{g_2^2}{2}\Big[W^+_\mu W^{+\mu} W^-_\nu W^{-\nu} - \left(W^+_\mu W^{-\mu}\right)^2 + 2W^+_\mu A^{3\mu} W^-_\nu A^{3\nu} - 2W^+_\mu W^{-\mu} A^3_\nu A^{3\nu}\Big]$$

Fermions

$$
\begin{aligned}
L &= \bar{\psi} i \gamma^\mu D_\mu \psi \\
D_\mu &= \partial_\mu - i g_2 A_\mu^a T^a - i g_1 Y B_\mu \\
&= \partial_\mu - i \frac{g_2}{\sqrt{2}} \left(W_\mu^+ T^+ + W_\mu^- T^- \right) \\
&\quad - ieQA_\mu - i\sqrt{g_1^2 + g_2^2} \left(T^3 - Q \sin^2 \theta_w \right) Z_\mu
\end{aligned}
$$

where

$$
e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \qquad Q = T^3 + Y
$$

1 generation of leptons

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \nu_R \quad e_R$$

$$Y = -\frac{1}{2} \qquad 0 \quad -1$$

ν_R does not interact

1 generation of leptons

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \nu_R \quad e_R$$
$$Y = -\frac{1}{2} \qquad 0 \quad -1$$

ν_R does not interact

Cannot write mass terms, but

$$L_y = -y\bar{l}_L \varphi e_R + \text{h.c.} = -\frac{y}{\sqrt{2}}(v+h)\left(\bar{e}_L e_R + \bar{e}_R e_L\right)$$

gives

$$m_e = \frac{yv}{\sqrt{2}}$$

and interaction with Higgs

$$-\frac{m_e}{v} h\bar{e}e$$

Neutrino mass

$$\tilde{\varphi}_i = \varepsilon_{ij}\varphi^i \qquad \tilde{\varphi} = \varphi^T \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$L_{y\nu} = -y_\nu \bar{\nu}_R \tilde{\varphi} l_L + \text{h.c.} = -\frac{y_\nu}{\sqrt{2}}(v+h)\left(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R\right)$$

$$m_\nu = \frac{y_\nu v}{\sqrt{2}}$$

1 generation of quarks

$$
\begin{array}{rccc}
q_L = & \begin{pmatrix} u_L \\ d_L \end{pmatrix} & u_R & d_R \\
Y = & -\frac{1}{6} & \frac{2}{3} & -\frac{1}{3}
\end{array}
$$

Color $SU(3)$, gluons

1 generation of quarks

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad u_R \quad d_R$$

$$Y = -\frac{1}{6} \qquad \frac{2}{3} \quad -\frac{1}{3}$$

Color $SU(3)$, gluons

$$L_y = -y_d \bar{q}_L \varphi d_R - y_u \bar{u}_R \tilde{\varphi} q_L + \text{h.c.}$$

$$m_u = \frac{y_u v}{\sqrt{2}} \qquad m_d = \frac{y_d v}{\sqrt{2}}$$

Interaction with Higgs

$$-\frac{m_u}{v} h \bar{u} u - \frac{m_d}{v} h \bar{d} d$$