Statistical properties of states in QED with unstable vacuum

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Plan of the talk

- QED with an unstable vacuum
- General density operators of QED states
- Reduced density operators
  - Subsystem of electrons or positrons
  - The von Neumann reduction
- Example: T-constant electric field

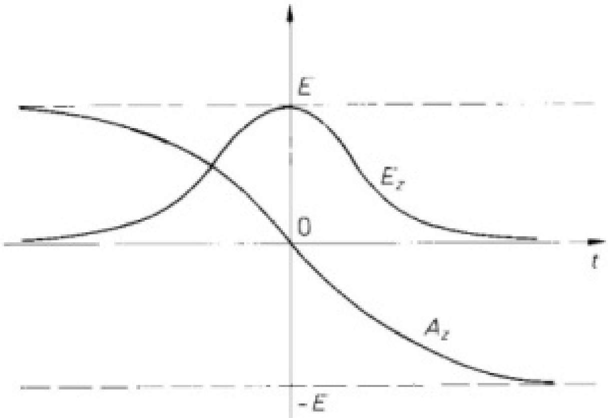
QED with unstable vacuum - 01

- QED with an unstable vacuum of the quantized Dirac or KG field (spinor in example).
  The Hamiltonian for spinor field $\psi(\mathbf{x})$ interacting with external electromagnetic field $A_{ext}(\mathbf{x},t)$ is formed in a standard way

$$H_e(t) = \int \bar{\psi}(\mathbf{x}) \left(-i\gamma\nabla + e\gamma A_{ext}(\mathbf{x},t) + m\right) \psi(\mathbf{x}) d\mathbf{x}$$

- Example of external field

$$\bar{E} = \left(0, 0, E\cosh^{-2} t\right), \quad A_\mu = \left(0, 0, 0, -E\tanh t\right)$$



QED with unstable vacuum - 03

- The Hamiltonians at t_{in} and t_{out} are different:

$$\begin{aligned} H_e(t_{in})_{\pm}\varphi(\mathbf{x}) &= {}_{\pm}\varepsilon(t_{in})_{\pm}\varphi(\mathbf{x}), \\ H_e(t_{out})^{\pm}\varphi(\mathbf{x}) &= {}^{\pm}\varepsilon(t_{out})^{\pm}\varphi(\mathbf{x}), \end{aligned}$$

- One can define in a common way a set of creation and destruction operators for electrons and positrons $a_n^{\dagger}(t_{in})$, $a_n(t_{in})$, $b_n^{\dagger}(t_{in})$, $b_n(t_{in})$ and $a_n^{\dagger}(t_{out})$, $a_n(t_{out})$, $b_n^{\dagger}(t_{out})$, $b_n(t_{out})$ which obey common commutation relations

$$\begin{aligned} [a_n(t_{in}), a_m^{\dagger}(t_{in})]_+ &= [a_n(t_{out}), a_m^{\dagger}(t_{out})]_+ = \\ [b_n(t_{in}), b_m^{\dagger}(t_{in})]_+ &= [b_n(t_{out}), b_m^{\dagger}(t_{out})]_+ = \delta_{mn} \end{aligned}$$

and all the others are zero.

QED with unstable vacuum - 04

- The Hamiltonians in terms of $a_n^\dagger(t_{in})$, $a_n(t_{in})$, $b_n^\dagger(t_{in})$, $b_n(t_{in})$ and $a_n^\dagger(t_{out})$, $a_n(t_{out})$, $b_n^\dagger(t_{out})$, $b_n(t_{out})$ are diagonal:

$$\begin{aligned} H_e(t_{in}) &= \sum_n \left[{}_+\varepsilon_n a_n^\dagger(t_{in}) a_n(t_{in}) + {}_- \varepsilon_n b_n^\dagger(t_{in}), b_n(t_{in}) \right], \\ H_e(t_{out}) &= \sum_n \left[{}^+\varepsilon_n a_n^\dagger(t_{out}) a_n(t_{out}) + {}^- \varepsilon_n b_n^\dagger(t_{out}), b_n(t_{out}) \right], \end{aligned}$$

- The corresponding vacuums are

$$\begin{aligned} a_n(t_{in})|0, t_{in}\rangle &= b_n(t_{in})|0, t_{in}\rangle = 0, \\ a_n(t_{out})|0, t_{out}\rangle &= b_n(t_{out})|0, t_{out}\rangle = 0 \end{aligned}$$

- Probability amplitude of transition from initial to final state

$$\begin{aligned} M_{in\to out} &= \langle t_{out}|U(t_{out}, t_{in})|t_{in}\rangle = \\ &= \langle 0, t_{out}| \cdots a_n(t_{out}) U(t_{out}, t_{in}) a_n^\dagger(t_{in}) \cdots |0, t_{in}\rangle. \end{aligned}$$

QED with unstable vacuum - 05

Moving to Heizenberg representation in a standard way:

- initial state operators $a_n^\dagger(\mathrm{in})$, $a_n(\mathrm{in})$ of electrons, and $b_n^\dagger(\mathrm{in})$, $b_n(\mathrm{in})$ of positrons; initial vacuum state

$$a_n(\mathrm{in})|0,\mathrm{in}\rangle = b_n(\mathrm{in})|0,\mathrm{in}\rangle = 0;$$

- final state operators $a_n^\dagger(\mathrm{out})$, $a_n(\mathrm{out})$, of electrons, and $b_n^\dagger(\mathrm{out})$, $b_n(\mathrm{out})$ of positrons; final vaccum state

$$a_n(\mathrm{out})|0,\mathrm{out}\rangle = b_n(\mathrm{out})|0,\mathrm{out}\rangle = 0;$$

- The probability amplitude for transition from an initial to a final state $M_{\mathrm{in}\to\mathrm{out}}$:

$$M_{\mathrm{in}\to\mathrm{out}} = \langle\mathrm{out}|\mathrm{in}\rangle = \langle 0,\mathrm{out}|\cdots a_n(\mathrm{out})a_n^\dagger(\mathrm{in})\cdots|0,\mathrm{in}\rangle.$$

- Operators $a_n^\dagger$, a_n, $b_n^\dagger$, b_n obey common commutation relations

$$\begin{aligned} &\left[a_m(in), a_n^\dagger(in)\right]_\pm = \left[a_m(out), a_n^\dagger(out)\right]_\pm \\ = \quad &\left[b_m(in), b_n^\dagger(in)\right]_\pm = \left[b_m(out), b_n^\dagger(out)\right]_\pm = \delta_{mn} \end{aligned}$$

QED with unstable vacuum - 06

Moving to Heizenberg representation in a standard way:

- The elementary probability amplitudes:

$$w\left(+|+\right)_{mn}=c_v^{-1}\langle 0,\mathrm{out}\left|a_m(\mathrm{out})a_n^\dagger(\mathrm{in})\right|0,\mathrm{in}\rangle,$$
$$w\left(-|-\right)_{nm}=c_v^{-1}\langle 0,\mathrm{out}\left|b_m(\mathrm{out})b_n^\dagger(\mathrm{in})\right|0,\mathrm{in}\rangle\,,$$
$$w\left(0|-+\right)_{nm}=c_v^{-1}\langle 0,\mathrm{out}\left|b_n^\dagger(\mathrm{in})a_m^\dagger(\mathrm{in})\right|0,\mathrm{in}\rangle\,,$$
$$w\left(+-|0\right)_{mn}=c_v^{-1}\langle 0,\mathrm{out}\left|a_m(\mathrm{out})b_n(\mathrm{out})\right|0,\mathrm{in}\rangle\,.$$

- Bogolubov transformation connecting the sets of *in*- and *out*-operators:

$$a(\mathrm{out})=\left[w\left(+|+\right)^\dagger\right]^{-1}a(\mathrm{in})-\kappa w\left(+-|0\right)\left[w\left(-|-\right)\right]^{-1}b^\dagger(\mathrm{in}),$$
$$b^\dagger(\mathrm{out})=\left[w\left(+|+\right)^\dagger\right]^{-1}w\left(+-|0\right)^\dagger a(\mathrm{in})+\left[w\left(-|-\right)\right]^{-1}b^\dagger(\mathrm{in}).$$

This relation is given by an unitary operator V:

$$V\left\{a\left(\mathrm{out}\right),\cdots\right\}V^\dagger=\left\{a\left(\mathrm{in}\right),\cdots\right\},\quad |0,\mathrm{in}\rangle=V|0,\mathrm{out}\rangle,\ V^\dagger V=I.$$

QED with unstable vacuum - 07

The state of the system is described by density operator (density matrix)

- The density operator of the system in terms of in-operators

$$\check{\rho}\left(\mathrm{in}\right) = \rho\left(a^{\dagger}(\mathrm{in}), a(\mathrm{in}), b^{\dagger}(\mathrm{in}), b(\mathrm{in})\right).$$

- The density operator in terms of out-set of the operators

$$\check{\rho}(\mathrm{out}) = V^{\dagger}\check{\rho}\left(\mathrm{in}\right) V .$$

We were interested in the density operator of the system that

- has vacuum initial state (vacuum at t_{in})
- has thermal initial state (system was in thermal equilibrium at t_{in})

General density operators - 01

The generating operator $\check{R}(J)$ that allows one to construct density operators $\check{\rho}$ with different initial conditions can be introduced [1]:

$$\check{R}(J) = Z^{-1}(J)\underline{\check{R}}(J),\ \mathrm{tr}\check{R}(J) = 1,\ Z(J) = \mathrm{tr}\underline{\check{R}}(J)\ ,$$

$$\underline{\check{R}}(J) = :\exp\left[\sum_n \left[a_n^\dagger(\mathrm{in})\left(J_{n,+}-1\right)a_n(\mathrm{in}) + b_n^\dagger(\mathrm{in})\left(J_{n,-}-1\right)b_n(\mathrm{in})\right]\right]:\ ,$$

- Vacuum initial case $J_{n,\zeta} = 0,\ \check{R}(J=0) = \check{\rho}(0)$ [2]:

$$\check{\rho}(0) = :\exp\left\{-\sum_n \left[a_n^\dagger(\mathrm{in})\ a_n(\mathrm{in})\ +\ b_n^\dagger(\mathrm{in})\ b_n(\mathrm{in})\right]\right\}:\ = |0,\mathrm{in}\rangle\langle 0,\mathrm{in}|.$$

- Thermal initial state $J = J_{n,\zeta}(\beta) = e^{-E_{n,\zeta}},\ \ E_{n,\zeta} = \beta\left(\varepsilon_{n,\zeta} - \mu_\zeta\right)$, $\check{R}(J = e^{-E_{n,\zeta}}) = \check{\rho}(\beta)$ [3]:

$$\check{\rho}(\beta) = Z_{gr}^{-1}\exp\left[-\beta\left(\check{H} - \sum_\zeta \mu_\zeta \check{N}_\zeta\right)\right],$$

$$Z_{gr} = \exp\left[\kappa\sum_{n,\zeta}\ln\left(1+\kappa e^{-E_{n,\zeta}}\right)\right].$$

General density operators - 02

Using the Bogolubov transformation, it is possible to present the generating operator $\check{R}(J)$ in terms of *out*-operators:

$$\check{R}(J) = Z^{-1}(J)\left|c_{\rm v}\right|^2 \det\left(1+\kappa AB\right)^{\kappa}\underline{\check{R}}(J),$$
$$\underline{\check{R}}(J) = :\exp\Big[-a^{\dagger}(\mathrm{out})\left(1-D_{+}\right)a(\mathrm{out}) - b^{\dagger}(\mathrm{out})\left(1-D_{-}\right)b(\mathrm{out})$$
$$-a^{\dagger}(\mathrm{out})C^{\dagger}b^{\dagger}(\mathrm{out}) - b(\mathrm{out})Ca(\mathrm{out})\Big]:, \quad \mathbb{J}_{mn,\zeta} = \delta_{mn}J_{n,\zeta},$$
$$D_{+} = w\left(+|+\right)\left(1+\kappa AB\right)^{-1}\mathbb{J}_{+}w\left(+|+\right)^{\dagger}, \; B = \kappa w\left(0|-+\right)$$
$$D_{-}^{T} = w\left(-|-\right)^{\dagger}\mathbb{J}_{-}\left(1+\kappa BA\right)^{-1}w\left(-|-\right), \quad A\left(J\right) = \mathbb{J}_{+}B^{\dagger}\mathbb{J}_{-},$$
$$C = w\left(-|-\right)^{\dagger}\mathbb{J}_{-}B\left(1+\kappa AB\right)^{-1}\mathbb{J}_{+}w\left(+|+\right)^{\dagger} + \kappa w\left(+-|0\right)^{\dagger},$$

- Example: obtaining explicit form of density operator for vacuum initial state $J_{n,\zeta} = 0$:

$$\check{\rho}\left(0\right) = \left|c_{\rm v}\right|^2 :\exp\Big\{-\sum_n\Big[a_n^{\dagger}(\mathrm{out})a_n(\mathrm{out}) + b_n^{\dagger}(\mathrm{out})b_n(\mathrm{out})$$
$$+\kappa a_n^{\dagger}(\mathrm{out})w\left(+-|0\right)_{nn}b_n^{\dagger}(\mathrm{out}) + \kappa b_n(\mathrm{out})w\left(+-|0\right)_{nn}^{\dagger}a_n(\mathrm{out})\Big]\Big\}: \; .$$

General density operators - 03

Knowing the explicit form of density operator, it is possible to calculate von Neumann entropy:

$$S(\check{\rho}) = -k_B \mathrm{tr} \check{\rho} \ln \check{\rho}.$$

- The entropy of vacuum initial state operator $\check{\rho}(0)$ – the state is pure

$$S(\check{\rho}(0)) = 0,$$

- The entropy of thermal initial state operator $\check{\rho}(\beta)$ – the state is mixed

$$\begin{aligned} S(\check{\rho}(\beta)) &= -k_B \sum_{n\zeta} \left\{ \kappa \left[1 - \kappa N_{n,\zeta}(\beta|\mathrm{in}) \right] \ln \left[1 - \kappa N_{n,\zeta}(\beta|\mathrm{in}) \right] \right. \\ &\left. + N_{n,\zeta}(\beta|\mathrm{in}) \ln N_{n,\zeta}(\beta|\mathrm{in}) \right\}. \end{aligned}$$

There is no difference if trace calculated either in terms of *in*- or in terms of *out*-operators, cause the evolution of the system is unitary.

Reduced density operators of QED states - 01

However, if we are interested only in one subsystem (or only one of them is availiable for observation), then we use the reduction procedure - we are averaging over one of subsystems

- Reduction procedure

$$\check{\rho}_{+} = \mathrm{tr}_{-}\check{\rho} = \sum_{M=0}^{\infty} \sum_{\{m\}} (M!)^{-1} \ {}_{b}\langle M_{\{m\}}, \mathrm{out}|\check{\rho}|M_{\{m\}}, \mathrm{out}\rangle_{b} \,,$$

$$\check{\rho}_{-} = \mathrm{tr}_{+}\check{\rho} = \sum_{M=0}^{\infty} \sum_{\{m\}} (M!)^{-1} \ {}_{a}\langle M_{\{m\}}, \mathrm{out}|\check{\rho}|M_{\{m\}}, \mathrm{out}\rangle_{a} \,,$$

where the states are given by

$$\begin{aligned} |M_{\{m\}}, \mathrm{out}\rangle_{b} &= (M!)^{-1/2}\, b^{\dagger}_{m_1}(\mathrm{out}) \ldots b^{\dagger}_{m_M}(\mathrm{out})|0, \mathrm{out}\rangle_{b}, \\ |M_{\{m\}}, \mathrm{out}\rangle_{a} &= (M!)^{-1/2}\, |a^{\dagger}_{m_1}(\mathrm{out}) \ldots a^{\dagger}_{m_M}(\mathrm{out})|0, \mathrm{out}\rangle_{a}. \end{aligned}$$

Reduced density operators of QED states - 02

The reduced density operators can be also obtained from reduced generating operators, which are calculated by taking partial trace over one of the subsystems:

$$\check{R}_{\pm}(J) = \mathrm{tr}_{\mp}\check{R}(J) \ ,$$

$$\check{R}_{+}(J) = Z_{+}^{-1}(J) \ :\exp\left\{-\sum_{n} a_n^{\dagger}(\mathrm{out})\left[1 - K_{+}(J)\right]_{nn} a_n(\mathrm{out})\right\}: ,$$

$$\check{R}_{-}(J) = Z_{-}^{-1}(J) \ :\exp\left\{-\sum_{n} b_n^{\dagger}(\mathrm{out})\left[1 - K_{-}(J)\right]_{nn} b_n(\mathrm{out})\right\}: ,$$

$$K_{\pm}(J) = D_{\pm} + C^{\dagger}\left(1 + \kappa D_{\mp}^{T}\right)^{-\kappa} C \, ,$$

$$Z_{\pm}^{-1}(J) = Z^{-1}(J)\left|c_{\mathrm{v}}\right|^{2} \det\left(1 + \kappa AB\right)^{\kappa} \det\left(1 + \kappa D_{\mp}\right)^{\kappa} \ .$$

Then one can ontain reduced density operators with different initial states by setting appropriate sources J.

Reduced density operators of QED states - 03

Reduced density operators of electron and positron subsystems with initial vacuum state can be obtained by setting $J = 0$ in $\check{R}_{\pm}(J)$ and have the form

$$\check{\rho}_{+}(0) = \check{R}_{+}(0) = |c_{\rm v}|^2 \; :\exp\left\{ -\sum_n a_n^{\dagger}(\mathrm{out}) \left[1 - \frac{P(+-|0)}{|c_{\rm v}|^2} \right]_{nn} a_n(\mathrm{out}) \right\}: ,$$

$$\check{\rho}_{-}(0) = \check{R}_{-}(0) = |c_{\rm v}|^2 \; :\exp\left\{ -\sum_n b_n^{\dagger}(\mathrm{out}) \left[1 - \frac{P(+-|0)}{|c_{\rm v}|^2} \right]_{nn} b_n(\mathrm{out}) \right\}: ,$$

where $P(+-|0)_{nn}$ is the probability of creation of pair with given quantum numbers n.

- Even if the initial state of the system was a pure state, these operators (and reduced operators in general) describe mixed states, and their entropy is different from the entropy of initial state.

von Neumann reduction - 01

The other possible source of entropy generation in the system is decoherence, which can occur in result of a measurement of a physical value by a classical instrument.

- Reduction of density operator due to the measurements of number of particles:

$$\check{\rho}_N = \sum_s \langle s, \mathrm{out}|\check{\rho}(0)|s, \mathrm{out}\rangle \check{P}_s, \quad \check{P}_s = |s, \mathrm{out}\rangle\langle s, \mathrm{out}|.$$

- Vacuum initial state operator $\check{\rho}\,(0)$ after vN-reduction due to the measurement of number of particles:

$$\check{\rho}_N = |c_{\mathrm{v}}|^2 \sum_f W_f \check{P}_f, \quad \sum_f = \sum_{M=0}^{\infty} \sum_{Z=1}^{M} \sum_{\{m,n\}}, \quad \check{P}_f = |f, \mathrm{out}\rangle\langle f, \mathrm{out}|,$$

$$W_f = \left|w\left(+-|0\right)_{n_1 n_1}\right|^{2m_1} \ldots \left|w\left(+-|0\right)_{n_Z n_Z}\right|^{2m_Z}, \quad m_1 + m_2 + \ldots + m_z = M,$$

$$|f, \mathrm{out}\rangle = \frac{\left[a^\dagger_{n_1}(\mathrm{out}) b^\dagger_{n_1}(\mathrm{out})\right]^{m_1}}{m_1!} \cdots \frac{\left[a^\dagger_{n_Z}(\mathrm{out}) b^\dagger_{n_Z}(\mathrm{out})\right]^{m_Z}}{m_Z!} |0, \mathrm{out}\rangle,$$

Entropy of reduced density operators of QFT states - 01

- Vacuum initial state

Expressing vacuum-to-vacuum transition probability $|c_{\mathrm{v}}|^2$ in terms of mean numbers of particles created by external field from vacuum $N_n(0|\mathrm{out})$ (see, for example, [6]) we obtained expression for the entropy of the reduced operators $\check{\rho}_{\pm}(0)$ to obtain

$$S\left(\check{\rho}_{\pm}(0)\right) = \sum_n S(\check{\rho}_{n,\pm}(0)), \quad S(\check{\rho}_{n,\pm}(0)) = -k_B \mathrm{tr}\check{\rho}_{n,\pm}(0) \ln \check{\rho}_{n,\pm}(0)$$
$$= -k_B \left[\kappa \left(1 - \kappa N_n \left(0|\mathrm{out}\right)\right) \ln \left(1 - \kappa N_n \left(0|\mathrm{out}\right)\right) + N_n \left(0|\mathrm{out}\right) \ln N_n \left(0|\mathrm{out}\right)\right].$$

- Thermal initial case

It is turned out that it is possible to do almost the same in thermal case and obtain the expression for the entropy of the reduced operators $\check{\rho}_{\pm}(\beta)$ of the system that was initially in thermal state

$$S\left(\check{\rho}_{\pm}(\beta)\right) = -k_B \sum_n S\left(\check{\rho}_{n,\pm}(\beta)\right), \quad S\left(\check{\rho}_{n,\pm}(\beta)\right) = -k_B \mathrm{tr}\check{\rho}_{n,\pm}(\beta) \ln \check{\rho}_{n,\pm}(\beta) =$$
$$= -k_B \left\{\kappa \left[1 - \kappa N_{n,\pm}\left(\beta|\mathrm{out}\right)\right] \ln \left[1 - \kappa N_{n,\pm}\left(\beta|\mathrm{out}\right)\right] + N_{n,\pm}\left(\beta|\mathrm{out}\right) \ln N_{n,\pm}\left(\beta|\mathrm{out}\right)\right\}.$$

Entropy of von Neumann-reduced density operators - 01

- Von Neumann reduction

The entropy of vN-reduced density operator $\check{\rho}_N$ which with initial vacuum state:

$$
\begin{aligned}
S(\check{\rho}_N) &= -k_B \,\mathrm{tr}\check{\rho}_N \ln \check{\rho}_N = -k_B \sum_n \mathrm{tr}\, \check{\rho}_{N,n} \ln \check{\rho}_{N,n} = \\
&= -k_B \sum_n \{\kappa \left[1 - \kappa N_n(0|\mathrm{out})\right] \ln \left[1 - \kappa N_n(0|\mathrm{out})\right] + N_n(0|\mathrm{out}) \ln N_n(0|\mathrm{out})\} \, .
\end{aligned}
$$

It is the same expression as for the entropy of reduced density operators. This indicaties that the classical measurement of a number of particles leads to the same information loss as the averaging over one of subsystems.

T-constant field - 01

The T-constant electric field in $d = D + 1$ dimentions is defined as

$$\mathbf{E} = (0, E(t), 0, ..., 0), \; E(t) = \begin{cases} 0, & -\infty < t \leq t_{\text{in}} \\ E > 0, & t_{\text{in}} < t < t_{\text{out}} \\ 0, & t_{\text{out}} \leq t < \infty \end{cases},$$

- There is no particle production after the time instant t_{out}, and mean numbers of particles $N_{n,\zeta}(\cdots|out)$ created in a given state $n = \mathbf{p}, r$ ($\mathbf{p}$ is a D-dimensional vector of momentum and r is spin) depend only on the time interval $T = t_{out} - t_{in}$.
- Electric field acting during the sufficiently long time T creates a considerable number of pairs only in a finite region in the momentum space.

$$|p_\perp| \leq \sqrt{eE}\left[\sqrt{eE}T\right]^{1/2}, \;\; -T/2 \leq p_1/eE \leq T/2,$$

(see [5, 6, 7, 8]).

T-constant field - 02

Number of particles created in vacuum initial state by T-constant field is, in fact, independent of field action duration T , and are the same as in the case of the constant uniform electric field [9, 10]:

$$N_n\left(0|\mathrm{out}\right) = e^{-\pi\lambda}, \quad \lambda = (p_\perp^2 + m^2)/eE \ .$$

Summation over quantum numbers turns into integration over the momentun **p**, and the spin just gives numerical factor $\gamma_{(d)}$:

$$\sum_n \to \frac{\gamma_{(d)} V}{(2\pi)^{d-1}} \int d\mathbf{p},$$

After the integration in Dirac case we get

$$S(\check{\rho}_{\pm}(0)) = \gamma_{(d)} k_B \frac{(eE)^{\frac{d}{2}}\, TV}{(2\pi)^{d-1}} A_{Dirac}\left(d, E_c/E\right),$$

where $E_c = m^2/c$ is the critical field.

- It is possible to estimate the entropy in strong $E_c/E \ll 1$, critical $E_c/E = 1$, and weak $E_c/E \gg 1$ field limits. For example, for a strong field with $d = 4$ we have $A_{Dirac}\left(4, 0\right) = \pi^2/6$, for the critical field, we have $A_{Dirac}\left(4, 1\right) \approx 0,22$. In the case of a weak field the entropy has a small value of the order of $(\pi E_c/E)\exp[-\pi E_c/E]$ for any d. For $d = 3$ the following estimations hold $A_{Dirac}\left(3, 0\right) \approx 0,93$, $A_{Dirac}\left(3, 1\right) \approx 0,2$; for $d = 2$ the factor $A\left(2, 0\right)$ is a value of order of 1, and $A\left(2, 1\right) = e^{-\pi}$.

T-constant field - 03

Number of particles created in vacuum initial state by T-constant field is the same for Dirac and Bose case:

$$N_n\left(0|\mathrm{out}\right) = e^{-\pi\lambda}, \quad \lambda = \left(p_\perp^2 + m^2\right)/eE \ .$$

After the integration in Klein-Gordon case we get

$$S(\check{\rho}_\pm(0)) = k_B \frac{(eE)^{\frac{d}{2}}\, TV}{(2\pi)^{d-1}} A_{K-G}\left(d, E_c/E\right),$$

which differs from Dirac expression only by absence of spin-factor $\gamma_{(d)}$ and different numerical coefficient.

- Estimations for different field strengths are the following: $A_{K-G}\left(4,0\right) \approx 2,21$, $A_{K-G}\left(4,1\right) \approx 0,22$; $A_{K-G}\left(3,0\right) \approx 1,78$, $A_{K-G}\left(3,1\right) \approx 0,2$; $A_{K-G}\left(2,0\right) \approx 1$, $A_{K-G}\left(2,1\right) \approx e^{-\pi}$. In the case of weak field the entropy is a small value of the order of $\left(\pi E_c/E\right)\exp\left[-\pi E_c/E\right]$ for any d.

Conclusions

- Explicit form of desity operators with different initial states were constructed;
- Explicit forms of reduced density operators for electron and positron subsystems, and the explicit form of the density operator with vacuum initial state after the measurement of number of particles were obtained;
- Entropy of these reduced density operator as a function of only mean numbers of particles of a final state was obtained;
- T-constant electric field was considered as an example. The entropy of reduced density operators is porportional to the time of field action T and strength of applied field in power $d/2$.

Thank you for your attention!

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