Hadronic vacuum polarization contribution to g-2 of the muon in the nonlocal model

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Task

1. Get numerical results for contributions of quark vacuum polarization and vector mesons to anomalous magnetic moment of the muon at LO of $1/N_c$–expansion.
2. In the framework of nonlocal chiral quark model derive expression and get numerical results for $1/N_c$–corrections to hadron vacuum polarization
3. Derive an expression for two–photon–quark vertex with propagator $S_G^{-1}(p) = A(p)\not{p} - B(p)$.

Outline

1. Introduction
   - About anomalous magnetic moment
   - Hadron contribution
   - Nonlocal chiral quark model
2. LO contribution
   - Quark vacuum polarization
   - Vector mesons contribution
   - Nemerical results
3. NLO contribution
   - Polarization operator
   - Quark self–energy
   - Numerical results

Outline

Anomalous magnetic moment of the muon — is one of the most precisely measured quantities in particle physics. Our interest in very high precision measurements is motivated by our eagerness to exploit the limits of our present understanding of nature and to find effects which cannot be explained by the established theory. The most precisely studied lepton is the electron, but the muon can also be explored with extreme precision. And because effects of new physics are proportional to the mass of studying lepton we assume that muon is about 40000 times more sensitive than electron.

QED general vertex form

$$\Gamma^{\mu}(p,p') = \gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2), \quad q = p' - p, \quad p^2 = p'^2 = m^2, \qquad (1.1)$$

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Gyromagnetic ratio and AMM

$$g = 2\left[F_1(0) + F_2(0)\right] = 2 + 2F_2(0) \Rightarrow a = F_2(0) = (g-2)/2 \tag{1.2}$$

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Value measured at experiment E821 (BNL)

$$a_{\mu}^{\exp} = 11659208.0(6.3) \cdot 10^{-10}. \tag{1.3}$$

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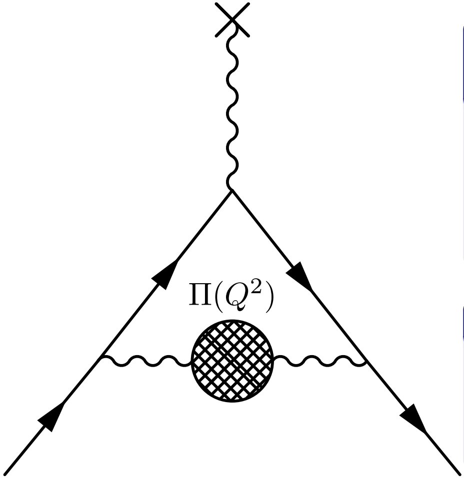
Standart model prediction

$$a_\mu^{\text{theory}} = 11659179.0(6.5) \cdot 10^{-10}. \tag{1.4}$$

Outline



Expression for $F_2(0)$ via polarization operator $\Pi(Q^2)$

$$F_2(0) = \frac{\alpha}{2\pi}\int_0^1 \mathrm{d}x \frac{(1-x)(2-x)}{x} D\left(\frac{m^2x^2}{1-x}\right) \tag{1.5}$$

Adler function definition

$$D(Q^2) = \frac{\mathrm{d}\Pi(Q^2)}{\mathrm{d}\ln(Q^2)} = Q^2\frac{\mathrm{d}\Pi(Q^2)}{\mathrm{d}Q^2} \tag{1.6}$$

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Bosonized action of SU(2)×SU(2)–symmetric chiral model

$$S = \int \mathrm{d}^4x \Bigg\{ \bar{q}(x) \left(i\partial\!\!\!/ - m_c + V\!\!\!/(x) + A\!\!\!/(x)\gamma_5 \right) q(x) - \frac{1}{2G_1}\left(\pi^a(x)^2 + \sigma(x)^2\right) + \frac{1}{2G_2}\left(\rho^{\mu a}(x)^2 + a_1^{\mu a}(x)^2\right) + \sum_{\Phi_i=\sigma,\pi,\rho,\omega,a_1} \Phi_i(x) \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 f(x_1) f(x_2) \bar{Q}(x-x_1,x)\Gamma_i Q(x,x+x_2) \Bigg\} \quad (1.7)$$

$$Q(x,y) = P\exp\left\{-i\int_x^y \mathrm{d}z^\mu (V_\mu^a + A_\mu^a(z)\gamma_5)T^a\right\} q(y), \quad (1.8)$$

$$\bar{Q}(x,y) = \bar{q}(x) P\exp\left\{-i\int_x^y \mathrm{d}z^\mu (V_\mu^a(z) - A_\mu^a(z)\gamma_5)T^a\right\}$$

Γ_i matrices for mesons

$$\Gamma_\sigma = 1, \quad \Gamma_\pi^a = i\gamma_5\tau^a, \quad \Gamma_\rho^{\mu a} = \gamma^\mu\tau^a, \quad \Gamma_\omega^\mu = \gamma^\mu, \quad \Gamma_{a_1}^{\mu a} = \gamma_5\gamma^\mu\tau^a, \tag{1.9}$$

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Gap equation

$$m(p) = m_{\mathrm{c}} + 4iG_1N_{\mathrm{f}}N_{\mathrm{c}}f^2(p)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{f^2(k)m(k)}{k^2-m^2(k)}, \quad S^{-1}(p) = \not{p} - m(p) \tag{1.10}$$

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Gap equation

$$m(p) = m_c + 4iG_1N_fN_cf^2(p)\int\frac{d^4k}{(2\pi)^4}\frac{f^2(k)m(k)}{k^2-m^2(k)}, \quad S^{-1}(p) = \not{p} - m(p) \tag{1.10}$$

Gap equation solution

$$m(p) = m_c - \sigma_0 f^2(p) = m_c + (m(0)-m_c)f^2(p) = m_c + m_d f^2(p), \tag{1.11}$$

Scalar Meson propagators

Bethe–Salpeter equation

$$D_{\sigma,\pi}(p) = \frac{1}{J_{\sigma,\pi}(p) - G_1^{-1}},\; J_{\sigma,\pi}(p) = i\int \frac{\mathrm{d}^4 k}{(2\pi)^4} f^2(k_-) f^2(k_+)\,\mathrm{tr}\left[S(k_-)\Gamma_{\sigma,\pi} S(k_+)\Gamma_{\sigma,\pi}\right] \tag{1.12}$$

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Vector mesons propagators

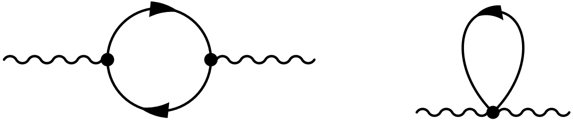
$$D^{\mu\nu}_{\omega,\rho,a_1}(p) = T^{\mu\nu} D^{\mathrm{T}}_{\omega,\rho,a_1}(p) + L^{\mu\nu} D^{\mathrm{L}}_{\omega,\rho,a_1}(p),\ D^{\mathrm{T,L}}_{\omega,\rho,a_1}(p) = \frac{1}{G_2^{-1} + J^{\mathrm{T,L}}_{\omega,\rho,a_1}(p)}$$

$$J^{\mu\nu}_{\omega,\rho,a_1}(p) = i\int \frac{\mathrm{d}^4k}{(2\pi)^4} f(k_-)^2 f(k_+)^2\,\mathrm{tr}\left[S(k_-)\Gamma^{\mu}_{\omega,\rho,a_1} S(k_+)\Gamma^{\nu}_{\omega,\rho,a_1}\right]. \quad (1.13)$$

Outline

Polarization operator

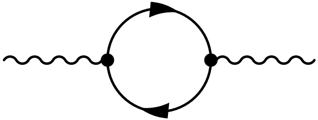
$$\Pi_{\mu\nu}(q) = iN_c \int \frac{d^4k}{(2\pi)^4} \operatorname{tr}\{\Gamma_\mu(k_+,k_-)S(k_+)\Gamma_\nu(k_+,k_-)S(k_-)\}$$
$$+ iN_c \int \frac{d^4k}{(2\pi)^4} \operatorname{tr}\{\Gamma_{\mu\nu}(k,q,-q)S(k)\}, \qquad (2.1)$$



(а) (б)

Transverse part of $\Pi_{\mu\nu}$ (A. E. Dorokhov Phys. Rev. D70, 2004, P. 094011)

$$\Pi(q^2) = \frac{4iN_c}{q^2}\int \frac{d^4k}{(2\pi)^4}\frac{\sum Q_i^2}{D_+D_-}\left\{m_+m_- - k_+k_- + \frac{2}{3}k_\perp^2 + \frac{4}{3}k_\perp^2 \times \left[m^{(1)}(k_+,k_-)^2(m_+m_- + k_+k_-) - m^{2(1)}(k_+,k_-)\right]\right\} + \frac{8iN_c}{q^2}\sum Q_i^2 \int \frac{d^4k}{(2\pi)^4}\frac{m_k}{D_k}\left[m_k' + \frac{4}{3}m^{(2)}(k,k,k+q)\right] \tag{2.2}$$



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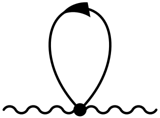

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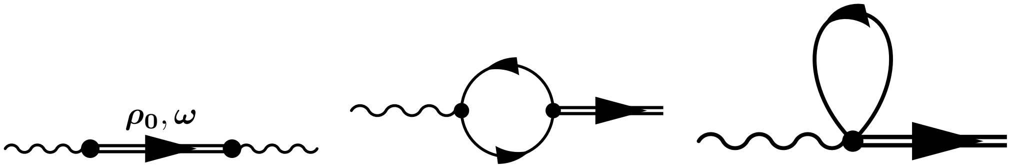
Outline

Photon–meson vertex

$$\Gamma^{\mu\nu}_{\gamma\to\rho_0,\omega}(q) = iN_c \int \frac{d^4k}{(2\pi)^4} \left\{ \mathrm{tr}\left[S_+\Gamma^\mu(k_+,k_-)S_-\Gamma^\nu_{\rho_0,\omega}(k_+,k_-)\right] + \mathrm{tr}\left[\Gamma^{\mu\nu}_{\rho_0,\omega}(k,q,-q)S_k\right]\right\}, \qquad (2.3)$$



(a) (б) (в)

Polarization operator (A. E. Dorokhov Phys. Rev. D70, 2004, P. 094011)

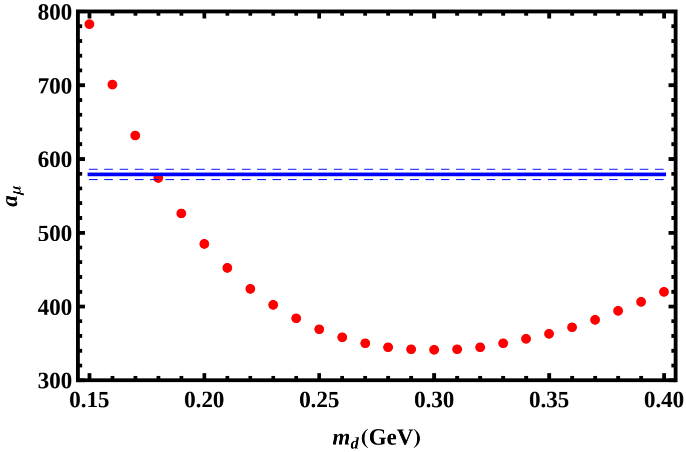
$$\Pi_{\rm v}(q) = \frac{1}{q^2}\frac{B_{\rm v}^2(q)}{G_2^{-1} - J^{\rm T}_{\rho_0,\omega}(q)} \tag{2.4}$$

$$B_{\rm v}(q) = 4iN_{\rm c}C_{\rho_0,\omega}\int\frac{{\rm d}^4k}{(2\pi)^4}\left[\frac{f_+f_-}{D_+D_-}\left(m_+m_- - k_+k_- + \frac{2}{3}k_\perp^2\left[1 - m^{2(1)}(k_+,k_-)\right]\right)\right.$$
$$\left. - \frac{4}{3}k_\perp^2\frac{f_k}{D_k}f^{(1)}(k,k+q)\right] \tag{2.5}$$

$C_{\rho_0,\omega} = {\rm tr}\left[(\tau^3 + 1/3)/2\cdot\tau^3\right] = 1$ for ρ_0–meson and ${\rm tr}\left[(\tau^3+1/3)/2\cdot 1\right] = 1/3$ for ω–meson. $C_{\rho_0,\omega}$ — trace over flavour space result.

Outline

Form–facotrs of nonlocal interaction $f(p)$ were chosen in Gauss form $f(p) = e^{-p^2/\Lambda^2}$.



Vector mesons contribution was calculated via

$$a_\mu = 4\alpha^2 \sum Q_i^2 \int_0^1 \mathrm{d}x(1-x)\Pi\left(\frac{m_\mu^2 x^2}{1-x}\right). \quad (2.6)$$

m_c MeV	m_d MeV	Λ GeV	G_1 GeV^{-2}	G_2 GeV^{-2}	$a_{\rho_0}\times 10^{10}$	$a_\omega \times 10^{10}$
7.6	300	1.04	32.7	−4.27	31	3.44
8.2	310	0.99	37.1	−5.23	34.3	3.81
8.8	320	0.95	41.8	−5.81	34.4	3.82
9.4	330	0.91	46.9	−6.14	32.7	3.63
10	340	0.87	52.4	−6.19	29.8	3.31
11	350	0.84	58.2	−5.87	25.5	2.84

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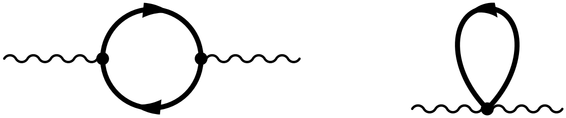
Polarization operator for quark propagator $S_G^{-1}(p) = A(p)\not{p} - B(p)$

$$\Pi_{\mu\nu}(q) = iN_c \int \frac{d^4k}{(2\pi)^4} \operatorname{tr}\left\{\Gamma^G_\mu(k_+, k_-) S_G(k_+) \Gamma^G_\nu(k_+, k_-) S_G(k_-)\right\} + iN_c \int \frac{d^4k}{(2\pi)^4} \operatorname{tr}\left\{\Gamma^G_{\mu\nu}(k, q, -q) S_G(k)\right\}, \tag{3.1}$$



(a) (б)

Dispersive part of polarization operator

$$
\begin{aligned}
\Pi_{\mathrm{D}}(q^2) = i\frac{N_{\mathrm{c}}}{q^2}\sum_i Q_i^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{1}{D_+^{\mathrm{G}} D_-^{\mathrm{G}}}\Bigg\{ & (A_+ + A_-)^2 (B_+ B_- - (k_+ k_-) A_+ A_-) + \frac{8}{3} k_\perp^2 A_+^2 A_-^2 \\
& + \frac{8}{3} k_\perp^2 \left[A^2 (k_+, k_-)^{(1)} (A_+ A_- [2k^2 - (k_+ k_-)] - B_+ B_-) - 2\frac{B_+^2 A_-^2 - B_-^2 A_+^2}{k_+^2 - k_-^2} \right] \\
& + \frac{16}{3} k_\perp^2 \Bigg[A_+ A_- (k_+ k_-) \left(B^{(1)}(k_+, k_-)^2 - k^2 A^{(1)}(k_+, k_-)^2 \right) \\
& + 2k^2 A^{(1)}(k_+, k_-) \left(A_+ A_- k^2 A^{(1)}(k_+, k_-) - \frac{A_- B_+^2 - A_+ B_-^2}{k_+^2 - k_-^2} \right) \\
& + B_+ B_- \left(B^{(1)}(k_+, k_-)^2 - k^2 A^{(1)}(k_+, k_-)^2 \right) \Bigg] \Bigg\}
\end{aligned} \tag{3.2}
$$

Contact part of polarization operator

$$\Pi_{\mathrm{C}}(q^2) = \frac{8iN_{\mathrm{c}}}{q^2}\sum_i Q_i^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{1}{D_k^{\mathrm{G}}}\left[\frac{2}{3}k_\perp^2 A_k A^{(1)}(k,k+q) + k^2 A_k A_k' + \frac{4}{3}k^2 k_\perp^2 A_k A^{(2)}(k,k+q,k) - B_k B_k' - \frac{4}{3}k_\perp^2 B_k B^{(2)}(k,k+q,k)\right] \quad (3.3)$$

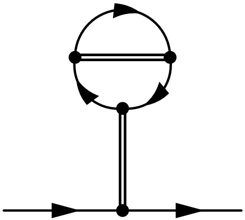
$$S_{\mathrm{G}}(p) = \frac{1}{\not{p} - m_p - \Sigma_p} = \frac{A_p \not{p} + B_p}{A_p^2 p^2 - B_p^2} = \frac{A_p \not{p} + B_p}{D_p^{\mathrm{G}}} \quad (3.4)$$

$$A_p = 1 + F_{\mathrm{v}}(p) \quad B_p = m_p + F_{\mathrm{s}}(p). \quad (3.5)$$

Outline

$1/N_c$ correction to quark self–energy

Sum of diagrams takes form $\Sigma_p = F_s(p) - F_v(p)\not{p}$



(a)

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Functions $F_\mathrm{v}(p)$ и $F_\mathrm{s}(p)$ depend on p^2 via (A. E. Radzhabov et al., Phys. Rev. D83, 2011, P. 116004)

$$F_\mathrm{s}(p) = i\sum_{\mathrm{M}=\sigma,\pi} f_p^2 \int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{f_{p-l}^2}{D_l^\mathrm{M}}\frac{\pm m_{p-l}}{D_{p-l}} - \frac{f_p^2}{D_0^\sigma}\sum_{\mathrm{M}=\sigma,\pi} i\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{1}{D_l^\mathrm{M}}$$
$$\times\, 4iN_\mathrm{c}N_\mathrm{f}\int\frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{f_k^2 f_{k+l}^4}{D_p D_{p+l}^2}\left[2k(k+l)m_{k+l} \pm m_k(m_{k+l}^2+(k+l)^2)\right].$$
$$F_\mathrm{v}(p) = -if_p^2\sum_{\mathrm{M}=\sigma,\pi}\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{f_{p-l}^2}{D_l^\mathrm{M}}\frac{1-(pl)/p^2}{D_{p-l}}. \tag{3.6}$$

Outline

Propagator parametrization (R. E. Arriola et al. Phys. Rev. D70, 2004, P. 097505)

$$S_{\mathrm{G}}(p) = \frac{Z(p)}{\not{p} - M(p)}, \quad Z(p) = A^{-1}(p), \quad M(p) = \frac{B(p)}{A(p)}. \tag{3.7}$$

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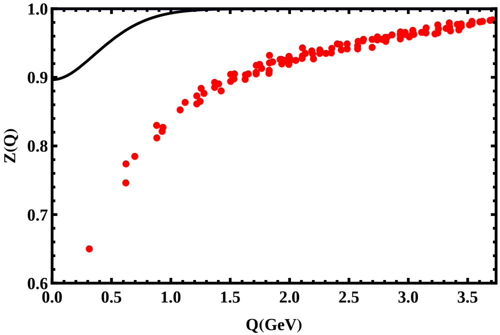

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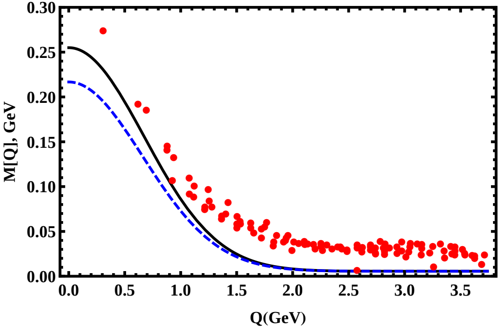

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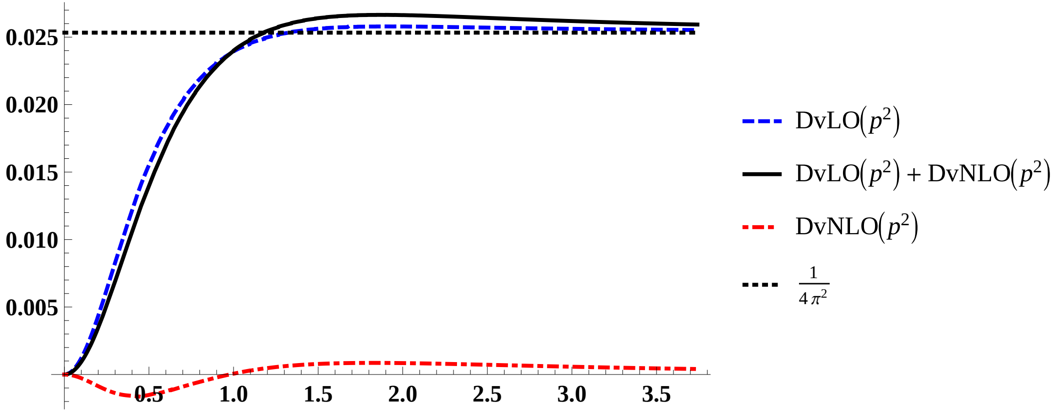
Adler function



Numerical results

m_c (MeV)	m_d (MeV)	Λ (GeV)	$a_{LO} \times 10^{10}$	$a_{LO+NLO} \times 10^{10}$
2.82	139.2	2.1	889	523
5.58	211.2	1.32	483	381
8.64	269.1	1	396	351
9.38	281.9	0.95	390	362
11.78	322.5	0.82	394	358
18.15	424	0.63	491	467

Conclusion

- $1/N_c$–corrections are relatively small and have different signs.
- To estimate all $1/N_c$–corrections we need to take into account other types of diagrams in this model.
- Numerical results do not strongly depend on parameter's chose or form of the nonlocal form–factor.