B decays

Andrey Grozin
A.G.Grozin@inp.nsk.su

Budker Institute of Nuclear Physics

B meson — QCD hydrogen atom

b sits at rest and creates chromoelectric field
Light constituents move in this external field
(relativistically; number not conserved)
Radius $1/\Lambda$, non-perturbative

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H^1 and H^3 — identical chemical properties
D and B — identical hadro-chemical properties

Spin symmetry

$\mu_p \sim e/(2m_p) \ll \mu_e$
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At $m \to \infty$ heavy-quark spin does not interact
Can be rotated without changing physics
$B \leftrightarrow B^*$ degenerate, identical properties

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Not only orientation but also magnitude is irrelevant
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Baryons with 2 heavy quarks — bound state $\sim 1/(m_Q \alpha_s)$
(like an antiquark with $s = 0, 1$)

$\bar{b}q$ mesons

$s_b = 0$:

S-wave $j^P = \frac{1}{2}^+$

P-wave $j^P = \frac{1}{2}^-, \frac{3}{2}^-$

Fine splitting $\sim \Lambda$

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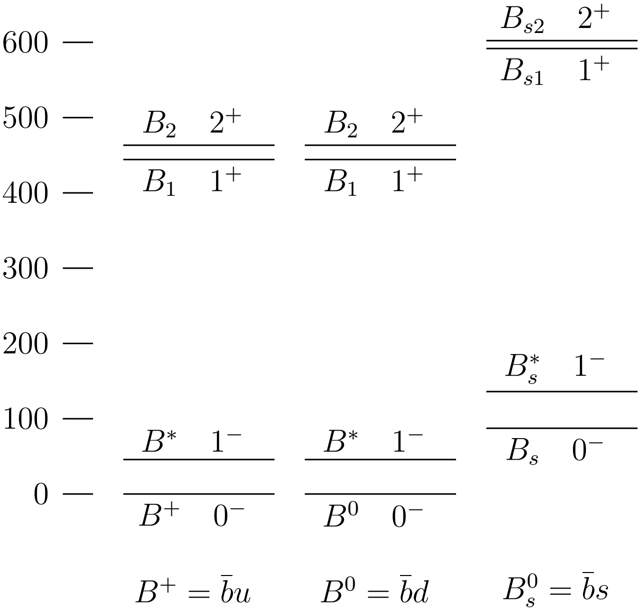
1^+ mesons: don't differ by exactly conserved quantum numbers; differ by j conserved up to $1/m_b$
$\Rightarrow$ mixing angle $\sim \Lambda/m_b$

B mesons


600
500
400
300
200
100
0
B_{s2} 2^+
B_{s1} 1^+
B_2 2^+
B_1 1^+
B_2 2^+
B_1 1^+
B_s^* 1^-
B_s 0^-
B^* 1^-
B^+ 0^-
B^* 1^-
B^0 0^-
$B^+ = \bar{b}u$
$B^0 = \bar{b}d$
$B_s^0 = \bar{b}s$

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$$\vec{s} = \vec{j} + \vec{s_b}$$

$$\vec{s}_b \cdot \vec{j} = \frac{1}{2}\left(s(s+1) - s_b(s_b+1) - j(j+1)\right) = \begin{cases} -\frac{3}{4} & s = 0 \\ \frac{1}{4} & s = 1 \end{cases}$$

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$$\frac{m_{B^*}^2 - m_B^2}{m_{D^*}^2 - m_D^2} = 1 \qquad \text{(experiment 0.89)}$$

P-wave excited states

$s_b = 0$

$\frac{1}{2}^- \to \frac{1}{2}^+ + \pi(0^-)\ \ l = 0 \Rightarrow \Gamma \sim p_\pi$

$\frac{3}{2}^- \to \frac{1}{2}^+ + \pi(0^-)\ \ l = 2 \Rightarrow \Gamma \sim p_\pi^5$

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$s_b = \frac{1}{2}$, $m_b = \infty$: $\Gamma(B_1) = \Gamma(B_2)$

- $B_1 \to B^*\pi$
- $B_2 \to B\pi,\ B^*\pi$

Shmushkevich factory

A sample of $j^P = \frac{3}{2}^-$ mesons, s_b randomly polarized
B_1: $\frac{3}{8}$; B_2: $\frac{5}{8}$

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B: $\frac{1}{4}$; B^*: $\frac{3}{4}$
B produced only from B_2

$$\frac{5}{8}B(B_2 \to B\pi) = \frac{1}{4}$$

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$$B(B_2 \to B\pi) = \frac{2}{5} \qquad B(B_2 \to B^*\pi) = \frac{3}{5}$$

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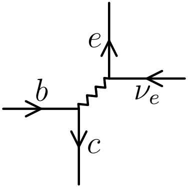
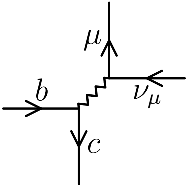
$$\frac{\Gamma(B_2 \to B\pi)}{\Gamma(B_2 \to B^*\pi)} = \frac{2}{3} \left(\frac{p_\pi(B_2 \to B\pi)}{p_\pi(B_2 \to B^*\pi)} \right)^5$$

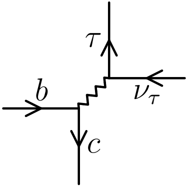
Semileptonic decays

$$\frac{G^2 m_b^5}{192\pi^3}|V_{cb}|^2\times$$





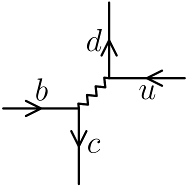
$$f\left(\frac{m_c}{m_b}\right)$$



$$f\left(\frac{m_c}{m_b},\frac{m_\tau}{m_b}\right)$$

Hadronic decays

$$\frac{G^2 m_b^5}{192\pi^3}|V_{cb}|^2 \times$$



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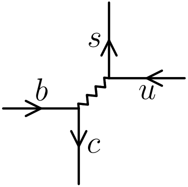
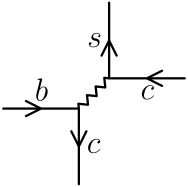


$$N_c f\left(\frac{m_c}{m_b}\right)$$



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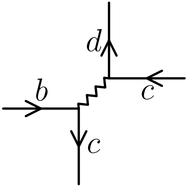

$$N_c f\left(\frac{m_c}{m_b}, \frac{m_c}{m_b}\right)$$

$$B(B \to X_c e \bar{\nu}_e) \approx \frac{1}{9}$$

Semileptonic width

$$\Gamma(b \to ul\bar{\nu}) = \frac{G^2 m_b^5}{193\pi^3} |V_{cb}|^2$$

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$$\Gamma(b \to u l \bar{\nu}) = \frac{G^2 m_b^5}{193\pi^3} |V_{cb}|^2 \left[1 - \frac{\mu_\pi^2 + 3\mu_G^2}{2m_b^2} \right]$$

μ_π^2 — relativistic time dilation

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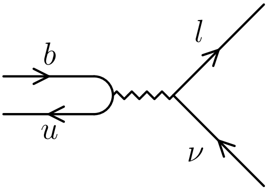
μ_π^2 — relativistic time dilation

$$\Gamma(b \to cl\bar{\nu}) = \frac{G^2 m_b^5}{193\pi^3}|V_{cb}|^2 \left[\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) f\left(\frac{m_c}{m_b}\right) + \frac{\mu_G^2}{m_b^2} f_2\left(\frac{m_c}{m_b}\right)\right]$$

α_s correction

$\bar{B}^- \to l^- \bar{\nu}_l$



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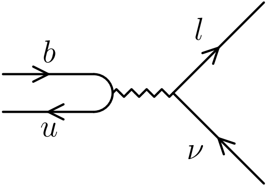

$$<0|\bar{u}\gamma^\mu\gamma_5 b|B> = i f_B p^\mu$$

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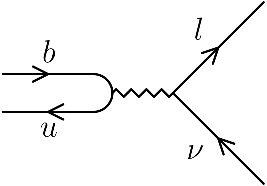

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Relativistic normalization

$$<B(\vec{p}\,')|B(\vec{p})> = (2\pi)^3 2p_0 \delta(\vec{p}\,' - \vec{p})$$

Nonrelativistic normalization

$$_{\rm nr}<B(\vec{p}\,')|B(\vec{p})>_{\rm nr} = (2\pi)^3 \delta(\vec{p}\,' - \vec{p})$$

$s_b = 0$

$$<0|\bar{u}b|B>_{\rm nr} = -iF\bar{v}$$

$\not{v}v = -v$, $\bar{v}v = -1$

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$s_b = \frac{1}{2}$ with a definite polarization u

$$<0|\bar{u}_i b^j|B>_{\rm nr} = -iF\bar{v}_i u^j$$

$$<0|\bar{u}\Gamma b|B>_{\rm nr} = -iF\,{\rm Tr}\,\Gamma M \qquad M = u\bar{v}$$

$\not{v}u = u$, $\bar{u}u = 1$

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$\not{v}u = u$, $\bar{u}u = 1$

$$\not{v}M = -M\not{v} = M$$

$${\rm Tr}\,\bar{M}M = (\bar{u}u)\,(\bar{v}v) = -1$$

Spin structures

$$M = \frac{1 + \not{v}}{2\sqrt{2}} \gamma_5 \qquad M^* = -\frac{1 + \not{v}}{2\sqrt{2}} \not{\epsilon}$$

Heavy-quark spin rotations act on the left index

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Heavy-quark spin rotations act on the left index

$$<0|\bar{u}\Gamma b|B> = \sqrt{2m_B}\,\mathrm{Tr}\,\Gamma M$$
$$<0|\bar{u}\Gamma b|B^*> = \sqrt{2m_{B^*}}\,\mathrm{Tr}\,\Gamma M^*$$

f_B

$$<0|\bar{u}\gamma^\mu\gamma_5 b|B> = -i\sqrt{2m_B}F\,\mathrm{Tr}\,\gamma^\mu\gamma_5 M = 2i\sqrt{m_B}Fv^\mu = if_B p^\mu$$

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$$<0|\bar{u}\gamma^\mu b|B^*> = -i\sqrt{2m_{B^*}}F\,\mathrm{Tr}\,\gamma^\mu M^* = 2i\sqrt{m_{B^*}}Fe^\mu = im_{B^*}f_{B^*}e^\mu$$

$$f_{B^*} = f_B$$

Heavy-quark spin symmetry

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Heavy-quark spin symmetry

$$F \sim \Lambda^{3/2} \qquad f_B \sim \frac{\Lambda^{3/2}}{m_B^{1/2}}$$

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}$$

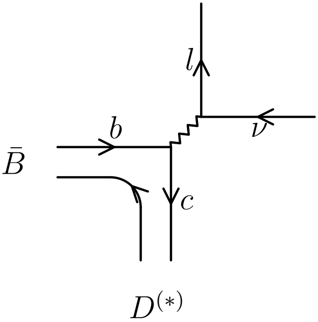
$\bar{B}^- \to l^- \bar{\nu}_l$

$$M = \frac{G}{\sqrt{2}} V_{ub} f_B p_B^\mu \, \bar{u} \gamma_\mu (1 + \gamma_5) v = \frac{G}{\sqrt{2}} V_{ub} f_B m_l \, \bar{u} (1 + \gamma_5) v$$

$$\Gamma = \frac{G^2}{128\pi^2} |V_{ub}|^2 f_B^2 m_B m_l^2$$

const at $m_B \to \infty$ $(\Gamma_{\text{tot}} \sim m_B^5)$

$\bar{B} \to D^{(*)} l \bar{\nu}$



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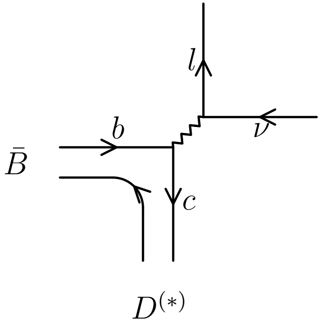

$s_b = s_c = 0$ $J = \bar{c}_{v'} b_v$

Counting form factors — brick wall frame

$$_{\rm nr}\langle D(v')|J|\bar{B}(v)\rangle_{\rm nr} = \xi(w)\,\bar{v}(v)u(v)$$

$$w = v \cdot v' = \cosh\vartheta \in \left[1, \frac{m_B^2 + m_D^2}{2m_B m_D}\right]$$

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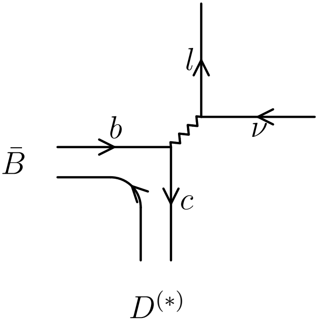

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$v' = v$ $(w = 1)$

$\xi(1) = 1$

$\bar{B} \to D^{(*)}$ form factors

$$<D|\bar{q}\Gamma b|\bar{B}> = -\sqrt{2m_B}\sqrt{2m_D}\xi(w)\,\mathrm{Tr}\,\bar{M}'\Gamma M$$

$$<D^*|\bar{q}\Gamma b|\bar{B}> = -\sqrt{2m_B}\sqrt{2m_{D^*}}\xi(w)\,\mathrm{Tr}\,\bar{M}^{*\prime}\Gamma M$$

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$$<D|\bar{q}\gamma^\mu b|\bar{B}> = \sqrt{m_B m_D}\xi(w)(v+v')^\mu$$
$$<D^*|\bar{q}\gamma^\mu b|\bar{B}> = i\sqrt{m_B m_{D^*}}\xi(w)\,i\varepsilon^{\mu\alpha\beta\gamma}e^*_\alpha v_\beta v'_\gamma$$
$$<D^*|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}> = \sqrt{m_B m_{D^*}}\xi(w)\,[(e^*\cdot v)v'^\mu - (1+w)e^{*\mu}]$$

$\bar{B} \to D^{(*)} l \bar{\nu}$

$$\frac{d\Gamma(\bar{B} \to D l \bar{\nu})}{dw} = \frac{G^2}{48\pi^3} |V_{cb}|^2 m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} \xi^2(w)$$

Why $(w^2 - 1)^{3/2}$?

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Why $(w^2 - 1)^{3/2}$?

$$\frac{d\Gamma(\bar{B} \to D^* l \bar{\nu})}{dw} = \frac{G^2}{4\pi^3} |V_{cb}|^2 m_{D^*}^3 (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} \xi^2(w)$$

Why $(m_B - m_{D^*})^2$?

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Why $(m_B - m_{D^*})^2$?

Luke theorem: no $1/m_c$ correction to $\bar{B} \to D^* l \bar{\nu}$ at $w \to 1$