On calculation of cross sections in Lorentz-violating QED

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based on hep-ph/1204.5782 + further work

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Outline

- Motivation of LV
- Model, modified Feynman rules
- Perturbative calculations:
  $\gamma \to e^+e^-, \quad e^- \to e^-\gamma, \quad \gamma\gamma \to e^+e^-, \quad \gamma N \to Ne^+e^-$
- Conclusions for experiments.

Motivation

- Several approaches to quantum gravity predict Lorentz-invariance violation (LIV) at high energies (e.g. Horava-Lifshitz).

  Horava, 2009

  Blas, Pujolas, Sibiryakov, 2010

- Theories with violation of the Lorentz Symmetry as an effective theory

  Kostelecky, Colladay, 1998

  Coleman, Glashow, 1999

- Lorentz invariance as an emergent phenomena at low energies

  Chadha, Nielsen, 1982

Model

Assumptions: $C-$, $P-$, $T-$parity, rotational symmetry, LV operators of dimension up to 6.

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} +$$

$$+ i\varkappa\bar{\psi}\gamma^{i}D_{i}\psi + \frac{ig}{M^2}D_{j}\bar{\psi}\gamma^{i}D_{i}D_{j}\psi + \frac{\xi}{4M^2}F_{kj}\partial_{i}^{2}F^{kj}$$

$$\Downarrow$$

Dispersion relations:

$$E_{\gamma}^{2} = k^2 + \frac{\xi k^4}{M^2}, \qquad E_{e}^{2} = m^2 + p^2\left(1 + 2\varkappa\right) + \frac{2gp^4}{M^2}.$$

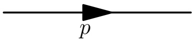
Modified spin sums and propagators

$$\sum_{a=1,2} \varepsilon_\mu^{(a)} \varepsilon_\nu^{(a)} = diag\left(-1 - \frac{\xi^2 k^2}{M^2}, 1, 1, 1\right)$$

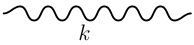
$$\sum_{s=1,2} u^s(p)\bar{u}^s(p) = \gamma^\mu \tilde{p}_\mu + m, \qquad \sum_{s=1,2} v^s(p)\bar{v}^s(p) = \gamma^\mu \tilde{p}_\mu - m$$

$$\tilde{p}^0 = E, \qquad \tilde{p}^i = p^i\left(1 + \varkappa + \frac{gp^2}{M^2}\right)$$

Propagators

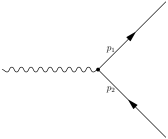


$$= \frac{i\left(\gamma^\mu \tilde{p}_\mu + m\right)}{\tilde{p}^\mu \tilde{p}_\mu - m^2 + i\epsilon}$$

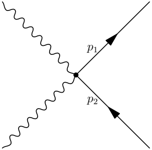


$$= \frac{i}{E^2 - k^2\left(1 + \frac{\xi k^2}{M^2}\right) + i\epsilon} diag\left(-1 - \frac{\xi k^2}{M^2}, 1, 1, 1\right)$$

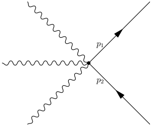
Modified vertices



$$= -ie\gamma^\mu - ie\delta_i^\mu \left[\varkappa\gamma^i + \frac{g}{M^2}\left(p_1^i(p_1\cdot\gamma + \right.\right.$$
$$\left.\left. + p_2^i(p_2\cdot\gamma) - (p_1\cdot p_2)\gamma^i\right)\right]$$



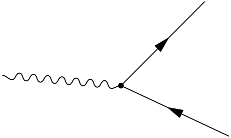
$$= \frac{ige^2}{M^2}\left[\gamma^i(p_2-p_1)^j + \gamma^j(p_2-p_1)^i + \delta^{ij}\left((p_2-p_1)\cdot\gamma\right)\right]\delta_i^\mu\delta_j^\nu$$



$$= -\frac{2ige^3}{M^2}\left[\delta_i^\mu\delta_j^\nu\delta_j^\lambda + \delta_j^\mu\delta_i^\nu\delta_j^\lambda + \delta_j^\mu\delta_j^\nu\delta_i^\lambda\right]$$

Photon decay $\gamma \to e^+e^-$ and Cerenkov radiation $e^- \to e^-\gamma$

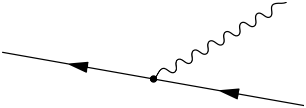
Photon decay



If $0 \le 2g \le \xi_*$ or $0 < g,\ g/2 \le \xi_*$,
$\left(\xi_* = \xi - 2M^2\varkappa/k^2\right)$.

$$\Gamma = \alpha k \left[-\frac{2\varkappa}{3} + \frac{k^2}{M^2}\left(\frac{\xi}{3} - \frac{11g}{30}\right)\right]$$

Cerenkov radiation



The case $0 \le \varkappa,\ 0 \le,\ \xi \le 2g$.

$$\Gamma = \alpha p \left[\frac{4\varkappa}{3} + \frac{p^2}{M^2}\left(\frac{157g}{60} - \frac{11\xi}{60}\right)\right]$$

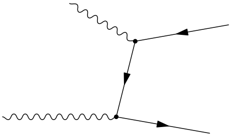
$$\frac{dE}{dt} = -\alpha p^2 \left[\frac{7\varkappa}{12} + \frac{p^2}{M^2}\left(\frac{11g}{12} - \frac{2\xi}{15}\right)\right]$$

The case of dim 4 operators $(\xi = g = 0)$ — *Klinkhamer, Schreck, 2008*

Hohensee et.al., 2008

Effects of LIV in matrix element and in phase volume are of the same order!

Pair production on the soft photon $\gamma\gamma \mapsto e^+e^-$



$$\omega_{LV} = -\varkappa k + \left(\frac{\xi}{2} - g\right)\frac{k^3}{M^2}.$$

If $k(2\omega + \omega_{LV}) \gg m^2$,

$$\sigma = \frac{\alpha^2\pi}{2k\omega}\left[1 + \left(1 + \frac{\omega_{LV}}{\omega}\right)^2\right]\ln\left[\frac{k(2\omega+\omega_{LV})}{m^2}\right]$$

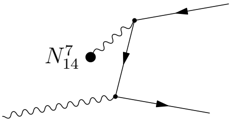
LI case:

$$\sigma = \frac{\alpha^2\pi}{2k\omega}\ln\left[\frac{2k\omega}{m^2}\right]$$

BUT: $\omega_{LV} > 0$ $\longrightarrow$ photon decay $\gamma \to e^+e^-$

thresholds — *Galaverni, Sigl, 2008*

Pair production on a nuclei $\gamma N \mapsto e^+ e^- N$



$$\omega_{LV} = -\varkappa k + \left(\frac{\xi}{2} - g\right)\frac{k^3}{M^2}, \qquad k \gg m$$

$$k\,|\omega_{LV}| \ll m^2 \quad \longrightarrow \quad \sigma = \frac{28}{9}\frac{z^2\alpha^3}{m^2}\ln\frac{2k}{m}$$

Bethe, Heitler, 1934

$$k\,|\omega_{LV}| \gg \frac{m^2}{\alpha^2 Z^{2/3}} \quad \longrightarrow \quad \sigma = \frac{4Z^2\alpha^3}{3}\frac{1}{k\,|\omega_{LV}|}\left(\ln\frac{k\,|\omega_{LV}|}{m^2}\right)^2$$

$$m^2 \ll k\,|\omega_{LV}| \ll \frac{m^2}{\alpha^2 Z^{2/3}} \longrightarrow \sigma = \frac{8Z^2\alpha^3}{3}\frac{1}{k\,|\omega_{LV}|}\ln\frac{2k\,|\omega_{LV}|}{m^2}\ln\frac{1}{\alpha Z^{1/3}}$$

Bounds on LV parameters

$$E_\gamma^2 = k^2 + \frac{\xi k^4}{M^2}, \qquad E_e^2 = m^2 + p^2\left(1 + 2\varkappa\right) + \frac{2gp^4}{M^2}.$$

- $|\varkappa| < 10^{-15}$ — absence of anomally synchrotron losses at LEP

  Altshul, 2009

- $-5 \cdot 10^{13} < \xi < 10^{11}$ — photons with energies 50 TeV have been detected

  HEGRA, CANGAROO, HESS

- $-10^{11} < g < 10^{6}$ — photon spectrum of Crab nebula and AGNs.(assumption — SSC model)

  Altshul, 2006

Possible future bounds

GZK cutoff: $p\,\gamma_{CMB} \;\rightarrow\; p\,\pi_0$ or $p\,\gamma_{CMB} \;\rightarrow\; n\,\pi_+$,

$$\pi_0 \rightarrow \gamma\,\gamma$$

If photon with energy $10^{19}\,\mathrm{eV}$ will be detected:

$-2\varkappa \cdot 10^{18} + \xi - \frac{g}{2} < 10^{-8}$ — γ-decay above threshold
$-2\varkappa \cdot 10^{18} + \xi - 2g > -10^{-8}$ — pp production decreases in 10 times

Model-independent bound!

Conclusions

- Effects of kinematical and dynamical aspects of LIV are of the same order
- Even small ($\varkappa \sim 10^{-26}$ or $\xi \sim g \sim 10^{-6}$) LIV strongly suppresses pair production (both in an electric field of a nuclei or in a weak magnetic field)
- Subplankian LIV is NOT experimentally closed yet.
- If even one photon with energy $10^{18} - 10^{19}$ eV will be detected, subplankian Lorentz violation will be strongly restricted.

Thank you for your attention!

Backup slides

Pair production in a weak magnetic field. LI case.

Semiclassical method: used to compute Shwinger effect (pp in electric field)

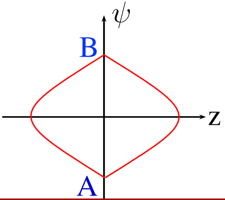
Affleck, Alvarez, Manton, 1982

Monin, Voloshin, 2008

$H = (H, 0, 0);\ k_\mu = (\omega, 0, \omega, 0)$

$$\Gamma \approx \int Dx_\mu\, e^{-S[x_\mu]}, \qquad S = \oint d\phi \left(m\sqrt{\dot{x}_\mu^2} - ieA_\mu \dot{x}_\mu \right) - \omega T + \omega Y,$$

$$\text{EOM: } \frac{m\ddot{x}_\mu}{\sqrt{\dot{x}_\mu^2}} = ieF_{\mu\nu}\dot{x}_\nu$$



Solution (the right arc):

$$t = \frac{\sqrt{p^2+m^2}}{eH}\left(\phi - \frac{\theta}{2}\right), \quad y = -i\psi = -i\frac{p}{eH}\mathrm{sh}\left(\phi - \frac{\theta}{2}\right),$$

$$x = 0, \qquad z = -\frac{p}{eH}\left[\mathrm{ch}\left(\phi - \frac{\theta}{2}\right) - \mathrm{ch}\frac{\theta}{2}\right]$$

Pair production in a weak magnetic field.

LI case

$$S=8\frac{}{3\frac{m^3}{\omega eH}, \qquad \Gamma\propto e^{-\frac{8}{3}\frac{m^3}{\omega eH}}}$$

Erber, 1966

LIV case

$$\beta = \varkappa - \frac{\xi}{2}\frac{k^2}{M^2} + \frac{2m^2}{\omega^2}$$

$$S = \frac{\omega^2}{eH}\frac{2\sqrt{2}}{3}\beta^{3/2}, \qquad \Gamma \propto e^{-S}.$$

$\beta > \frac{2m^2}{\omega^2}$ $\longrightarrow$ strong suppression

$0 < \beta < \frac{2m^2}{\omega^2}$ $\longrightarrow$ Γ grows — fine-tuning! $\left(\frac{2m^2}{\omega^2} \sim 10^{-26}\right)$