Effective phantom dark energy in scalar-tensor gravity

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Dark Energy equation of state

Null Energy Condition:

$$T_{\mu\nu} n^\mu n^\nu \geq 0$$

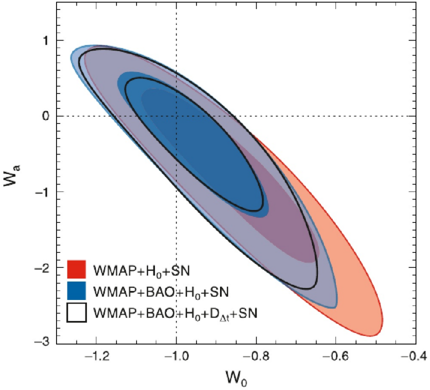
for every null vector n

For the perfect fluids it means: $\rho + p \geq 0$

$$w = p/\rho \geq -1$$

Observation data

$$w = w_0 + w_a(1-a) = w_0 + w_a\frac{z}{1+z}$$



$$w_0 = -0.93 \pm 0.13$$
$$w_a = -0.41 \pm 0.72$$
$$(68\% CL)$$

Expression for w_{eff}

Fridman equations:

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{eff})$$
$$\dot{H} = -4\pi G(\rho_m + \rho_{eff} + p_{eff})$$

Lead to general exprasion for $w_{eff} = p_{eff}/\rho_{eff}$:

$$w_{eff} = -\frac{1}{1-\Omega_m}\left(1 + \frac{2}{3}\frac{\dot{H}}{H^2}\right)$$

Brans-Dicke scalar-tensor gravity

Action in Jordan frame:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, [F(\Phi)R - Z(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - 2U(\Phi)] + S_m(\psi, g_{\mu\nu})$$

$$W_{BD} = \frac{F}{\left(\frac{dF}{d\Phi}\right)^2}.$$

Experimental constraints

Deviations from General Relativity

$$W_{BD} = \frac{F}{\left(\frac{dF}{d\Phi}\right)^2}; \quad W_{BD,0} > 4 \cdot 10^4$$

Time-variance of gravitational constant

$$8\pi G_{loc} = \frac{1}{F}\left(\frac{2F + 4(dF/d\Phi)^2}{2F + 3(dF/d\Phi)^2}\right); \quad \left(\frac{\dot{G}_{loc}}{HG_{loc}}\right)_0 < 0.5 \cdot 10^{-2}.$$

Constraints from BBN*

$$\frac{\Delta G_{cosm}}{G_{cosm}} \lesssim 0.1.$$

Cosmological equations

We consider flat Universe:

$$ds^2 = -dt^2 + a^2(t)dx^i dx^i.$$

And obtain the set of equations:

$$3FH^2 = \rho + \frac{1}{2}\dot{\Phi}^2 - 3H\dot{F} + U$$

$$-2F\dot{H} = \rho + \dot{\Phi}^2 + \ddot{F} - H\dot{F}$$

$$\ddot{\Phi} + 3H\dot{\Phi} = 3(\dot{H} + 2H^2)\frac{dF}{d\Phi} - \frac{dU}{d\Phi}.$$

$$\dot{\rho} + 3H\rho = 0$$

Expansion of functions

In series by redshift:

$$
\begin{aligned}
F(z) &= 1 + F_1 z + \frac{1}{2} F_2 z^2 + \frac{1}{6} F_3 z^3 \dots \\
U(z)/3H_0^2 &= \Omega_{U,0} + U_1 z \\
H^2(z)/H_0^2 &= 1 + h_1 z + \frac{1}{2} h_2 z^2 + \dots \\
\Phi'(z) &= \Phi_0' z + \frac{1}{2} \Phi_0'' z^2 \\
\rho(z)/3H_0^2 &= \Omega_{m,0}(1+z)^3 .
\end{aligned}
$$

and field Φ:

$$
\begin{aligned}
F &= 1 + f_1 \Phi + \frac{1}{2} f_2 \Phi^2 + \frac{1}{3} f_3 \Phi^3 , \\
U &= u_0 + u_1 \Phi + \frac{1}{2} u_2 \Phi^2 .
\end{aligned}
$$

Bounds on first derivative of F

With the use of the constraint on $W_{BD,0}$, using the normalization $F_0 = 1$, we get very strong upper bound on f_1:

$$|f_1| < 0.5 \cdot 10^{-2}.$$

This means that the field Φ is presently near the extremum of the funcion $F(\Phi)$.

From evolution equations by switching to derivatives by redshift, one can obtain:

$$\Phi_0'^2 = 6(1 - \Omega_{U,0} - \Omega_{m,0} - F_1).$$

So $\Phi_0'^2 \lesssim 1$, and for F_1 we have:

$$|F_1| \lesssim 10^{-2}.$$

Effective phantomness I

From the basic equation for w_{eff}:

$$1 + w_{eff,0} = \frac{f_2(\Phi_0')^2 + 6(1 - \Omega_{m,0} - \Omega_{U,0}) + f_1 \Phi_0''}{3(1 - \Omega_{m,0})}.$$

By extracting the second derivative of the field we find

$$\Phi_0'' = \left(2 - \frac{h_1}{2}\right) \Phi_0' - u_1 .$$

So, there are essentially two parameters that could yield $1 + w_{eff,0} < 0$, namely, f_2 and u_1.

Contribution of the potential

The possible contribution of the derivative of the potential to the phantom effective equation of state is given by

$$\Delta_u(1 + w_{eff,0}) = -f_1 \frac{u_1}{3(1 - \Omega_{m,0})}.$$

It is strongly supressed by small f_1, so this contribution can be sizeable only if the potential $U(\Phi)$ is very steep today.

However, steep potential would lead to the rapid acceleration of the scalar field, so the phantom phase would be very short in the past.

Furthermore, the fast evolution of the scalar field together with large $u_1 = dU/d\Phi(z = 0)$ would imply rapid change in time of the effective dark energy density.

Effective phantomness II

The final expession for w_{eff}:

$$1 + w_{eff,0} = \frac{(1 + f_2)(\Phi_0')^2}{3(1 - \Omega_{m,0})}.$$

Thus, the phantom behaviour today is controlled entirely by $f_2 = d^2F/d\Phi^2(z = 0)$. To have $w_{eff} < -1$ at the present time, one requires that

$$f_2 < -1.$$

This implies that today the field Φ must be close to a relatively sharp maximum of the function $F(\Phi)$.

Time-variance of w_{eff}

$$w_1 = \frac{dw_{\mathrm{eff}}}{dz}(z=0).$$

Observationally, $|w_1|$ is not large. From our expansion we obtain

$$w_1 = w_{1,A} + w_{1,B},$$

where $w_{1,A}$ and $w_{1,B}$ are:

$$w_{1,A} = \frac{1}{3(1-\Omega_{m,0})}(F_3 - 6u_1),$$

$$w_{1,B} = \frac{1}{3(1-\Omega_{m,0})^2}\left[(1+5\Omega_{m,0})(F_2 - 6\Omega_{U,0} - 3\Omega_{m,0} + 6) - 9\Omega_{m,0}\right] +$$

$$+\frac{1}{3(1-\Omega_{m,0})}\left[12F_2 + \frac{3F_2^2}{2} + 9F_2\Omega_{U,0} - \frac{9}{2}F_2\Omega_{m,0} - 30\Omega_{U,0}\right.$$

$$\left. +24(1-\Omega_{m,0}) + 6 \right] - \frac{1}{3(1-\Omega_{m,0})^2}\left(-F_2 + 6\Omega_{U,0} + 3\Omega_{m,0} - 6\right)^2.$$

For some "natural" values of parameters

$$\Omega_{m,0} \approx 0.25;$$

$$F_2 \approx -1..-10;$$

we typically have:

$$w_{1,B} \lesssim -10.$$

So for w_1 to stay in experimental bounds ($w_1 \lesssim 1$), one should demand*:

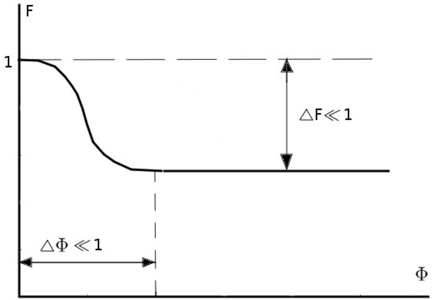
$$F_3 \gg 1$$

Shape of $F(\Phi)$

All obtained bound together imply that today the field Φ must be close to a relatively sharp maximum of the function $F(\Phi)$.
The "cosmological" gravity constant has not changed much since BBN.
So* the variation of $F(\Phi)$ has been small since BBN,

$$\Delta F \lesssim 0.1$$

Thus, the function $F(\Phi)$ must have the shape shown in



Redshift range of phantom stage

We can roughly estimate the range of redshifts in which the dynamics of F is non-trivial, and the phantom effective equation of state can be realized.

$$\frac{1}{2}(|f_2|\Phi_0')^2 z_{max}^2 \lesssim \Delta F$$

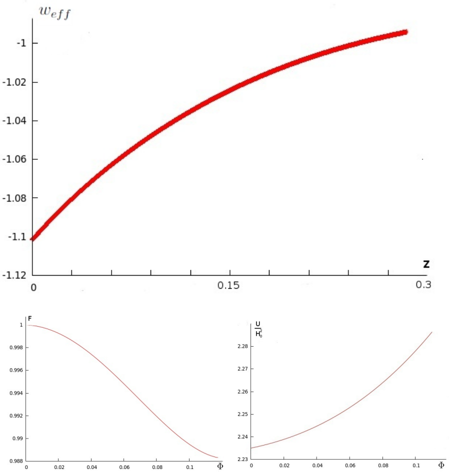
Let us denote by ϵ the deviation of w_{eff} from -1 today:

$$1 + w_{eff,0} = -\epsilon$$

It will bring us to the final expression for maximum redshift:

$$z_{max}^2 \lesssim \frac{2\Delta F}{3\epsilon} \frac{|1+f_2|}{|f_2|}.$$

Numerical example



Conclusions

- The problem of phantom behaviour in scalar-tensor theory revisited from a slightly different point of view
- It is possible to obtain and control effective phantom behavior even in simple scalar-tensor models, however one of the functions entering the scalar-tensor Lagrangian must have rather specific shape
- The redshift range of the phantom stage was roughly estimated and it is fairly small $z \lesssim 1$