QCD Sum Rules

Quantum Mechanic Models

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Overview

- Historical review
- QCD SR
- QM toy model
- QCD applications
- Condensates

Authors

QCD AND RESONANCE PHYSICS. THEORETICAL FOUNDATIONS

M.A. SHIFMAN, A.I. VAINSHTEIN * and V.I. ZAKHAROV
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(≈ 4000 citation)

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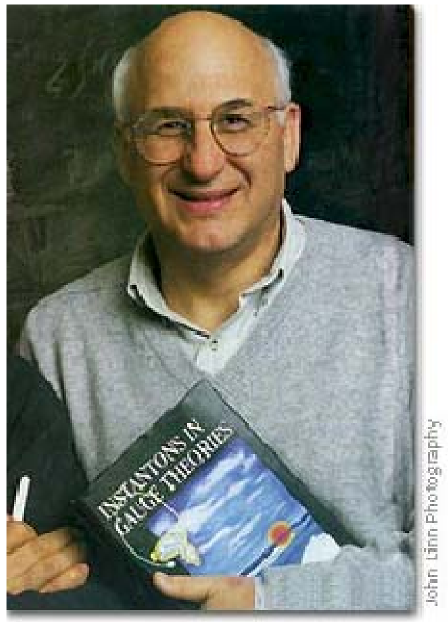
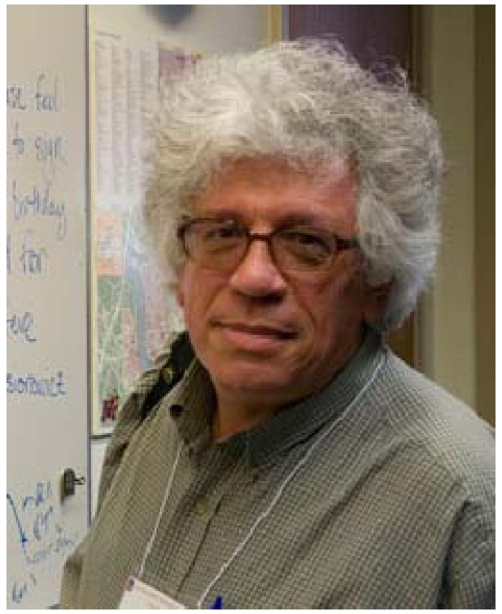
QCD Sum Rules was suggested in 1979 for studing meson properties by
Mikhail Shifman,   Arkady Vainshtein,   Valentin Zakharov





in trilogy papers QCD and Resonance Physics : Theoretical Foundation, Application, and The $\rho-\omega$ mixing.

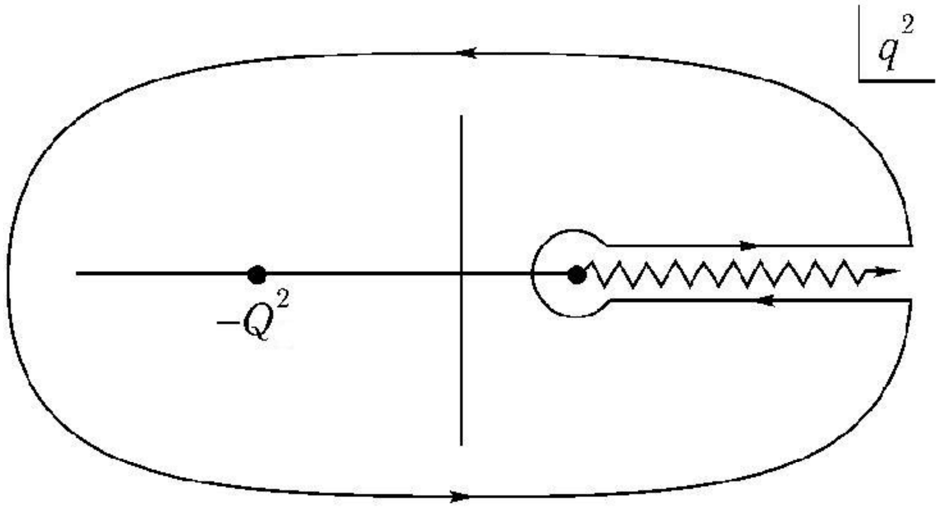
QCD Sum Rules

Determination of spectrum parameters from requirement of agreement between two ways for correlator $\Pi(Q^2 = -q^2)$ of quark currents : $\Pi(Q^2) = \int e^{-iqx} < 0 | J_1(0) J_2(x) | 0 > d^4 x$.

- Dispersion relation (Källén — Lehmann representation) and spectral representation:

$$\Pi(-q^2) = \int_{2m_\pi^2}^{\infty} \frac{\rho(s)}{s - q^2} ds + \text{subtract.},$$

$$\Pi(-q^2) = \sum_h \frac{<0|J_1|h><h|J_2|0>}{m_h^2 - q^2} + \text{subtract.}.$$



- 1 th way—Using dispersion relation with model spectral density $\rho_{had}(s)$ with parameters: decay constants f_h, masses m_h and others.

$$\rho_{had}(s) = f_h^2 \, \delta \left(s - m_h^2\right) + \rho_{pert}(s) \, \theta \left(s - s_0\right)$$

Quantum Mechanics

- Spectral representation after Borel summation:

$$M(\mu^2) \equiv B_{Q^2 \to \mu^2} \Pi(Q^2) = \sum_h \langle 0 | J_1 | h \rangle \langle h | J_2 | 0 \rangle e^{-m_h^2/\mu^2}.$$

This could me mimic by the sum in QM:

$$\mathrm{M}(\mu) = \sum_{k=0}^{\infty} |\Psi_k(0)|^2 e^{-E_k/\mu}.$$

where $\Psi_k(0)$ is eigenstate at origin and E_k energy level of k - state.

The function $\mathrm{M}(\mu) = \mathrm{G}(0, 1/i\mu \,|\, 0, 0)$ is related to two-time Green function in imaginary time:

$$\left[i \frac{\partial}{\partial t_2} - \hat{H}_2\right] G(\vec{x}_2, t_2 \,|\, \vec{x}_1, t_1) = \delta(t_2 - t_1)\, \delta(\vec{x}_2 - \vec{x}_1).$$

For D+1 Harmonic Oscillator model with mass $\mathbf{m}$, frequency ω, and radius vector $\mathbf{r}$, we have:

$$V(r) = \frac{m\,\omega^2}{2} r^2; \qquad E_k = \left(2k + \frac{D}{2}\right)\omega; \qquad |\Psi_k(0)|^2 = \left(\frac{m\omega}{\pi}\right)^{D/2} \frac{\left(k + \frac{D}{2} - 1\right)!}{\Gamma(D/2)\, k!}.$$

D+1 Harmonic Oscillator

For D+1 harmonic oscillator the exact solution is khown:

$$M^{osc}(\mu) = \left(\frac{m\,\omega}{2\,\pi\,\text{sh}\,(\omega/\mu)}\right)^{D/2}$$

The expansion of $M(\mu)$ at large μ coresspons to perturbative expansion of Green function:

$$G = \; G_0 + G_0 \otimes V \otimes G_0 + G_0 \otimes V \otimes V \otimes G_0 + \ldots,$$

and can be calulated to any order:

$$M^{pert}(\mu) \underset{\mu\to\infty}{\approx} M_0(\mu)\left(1 - \frac{D}{12}\left(\frac{\omega}{\mu}\right)^2 + \frac{D\,(4+5\,D)}{1440}\left(\frac{\omega}{\mu}\right)^4 + \ldots\right),$$

where function $M_0(\mu) = \left(\frac{m\,\mu}{2\,\pi}\right)^{D/2}$ is expression for free state ($V(r) = 0$) and can be expressed by:

$$M_0(\mu) = \int_0^\infty \rho_0(s)\, e^{-s/\mu}\, ds \quad \text{with spectral density } \rho_0(s) = \left(\frac{m\,s}{2\,\pi}\right)^{D/2} \frac{1}{s\,\Gamma(D/2)}\;.$$

Speaking in ***QCD*** language, free correlator $M_0(\mu)$ corresponds to perturbative contributions and the power correction $\left(\frac{\omega}{\mu}\right)^{n>0}$ to nonperturbative terms.

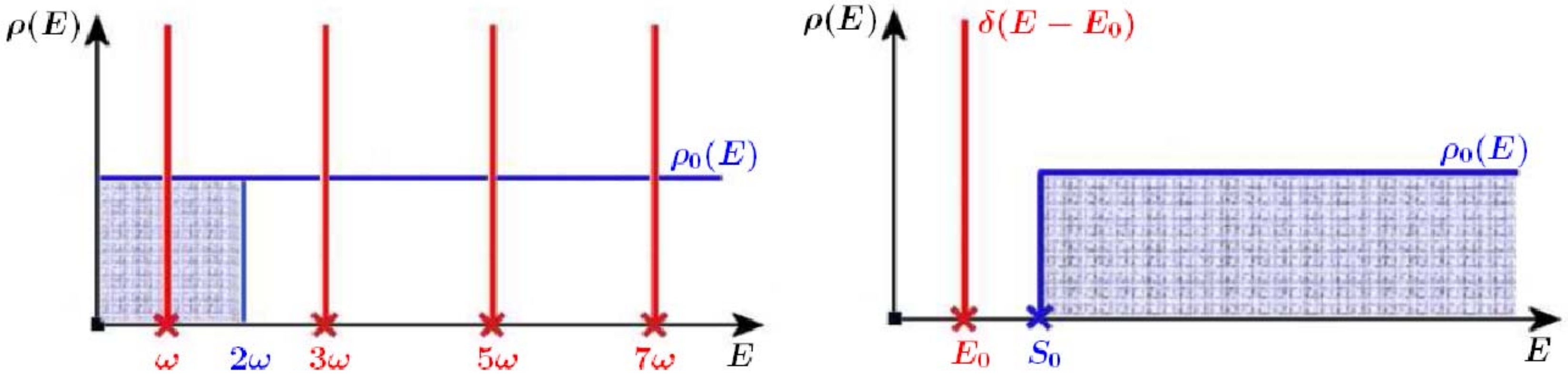
Model spectral density for harmonic oscillator

Form $M(\mu) = \sum_{k=0}^{\infty} |\Psi_k(0)|^2 e^{-E_k/\mu} = \int_0^{\infty} \rho^{exact}(s)\, e^{-s/\mu}\, ds$ we have

exact spectral density $\rho^{exact}(s) = \sum_{k=0}^{\infty} |\Psi_k(0)|^2 \delta(s - E_k)$,

model spectral density $\rho^{model}(s) = \sum_{k=0}^{n-1} |\Psi_k(0)|^2 \delta(s - E_k) + \Theta(s - s_0)\, \rho_0(s)$, thanks **Local Duality**.

n is number of states we want to study in SR (***n* = 1, 2**).



Exact vs Model spectral density with D=2 dimension.

QM SR for D+1 harmonic oscillator

Follow QCD, we keep only the couple “nonperturbative OPE” terms:

$$M^{OPE}(\mu) \approx \int_0^{\infty} \rho_0(s)\, e^{-s/\mu}\, ds + M_0(\mu)\left(-\frac{d}{12}\left(\frac{\omega}{\mu}\right)^2 + \frac{d\,(4+5\,d)}{1440}\left(\frac{\omega}{\mu}\right)^4 + \ldots\right)$$

Using model for spectral density with two states we have:

$$M^{spec}(\mu) \approx |\Psi_0(0)|^2\, e^{-E_0/\mu} + |\Psi_1(0)|^2\, e^{-E_1/\mu} + \int_{s_0}^{\infty} \rho_0(s)\, e^{-s/\mu}\, ds$$

where we have in general case 6 (4 for 2 states) parametes: $|\Psi_0(0)|^2$, $|\Psi_1(0)|^2$, E_0, E_1, s_0, μ.
Threshold s_0 is spectral model parameter and Borel parameter $\mu_{min} < \mu < \mu_{max}$ is free scale. The rest are physical characteristics, we extract form OPE in SR:

$$M^{spec}\left(\mu, s_0 \,\middle|\, |\Psi_0(0)|^2,\ |\Psi_1(0)|^2,\ E_0,\ E_1\right) \approx M^{OPE}(\mu), \text{ or explicitly}:$$

$$|\Psi_0(0)|^2\, e^{-E_0/\mu} + |\Psi_1(0)|^2\, e^{-E_1/\mu} = \int_0^{s_0} \rho_0(s)\, e^{-s/\mu}\, ds + M_0(\mu)\left(-\frac{D}{12}\left(\frac{\omega}{\mu}\right)^2 + \frac{D\,(4+5\,D)}{1440}\left(\frac{\omega}{\mu}\right)^4 + \ldots\right).$$

Resonances vs. "OPE" and exact solution

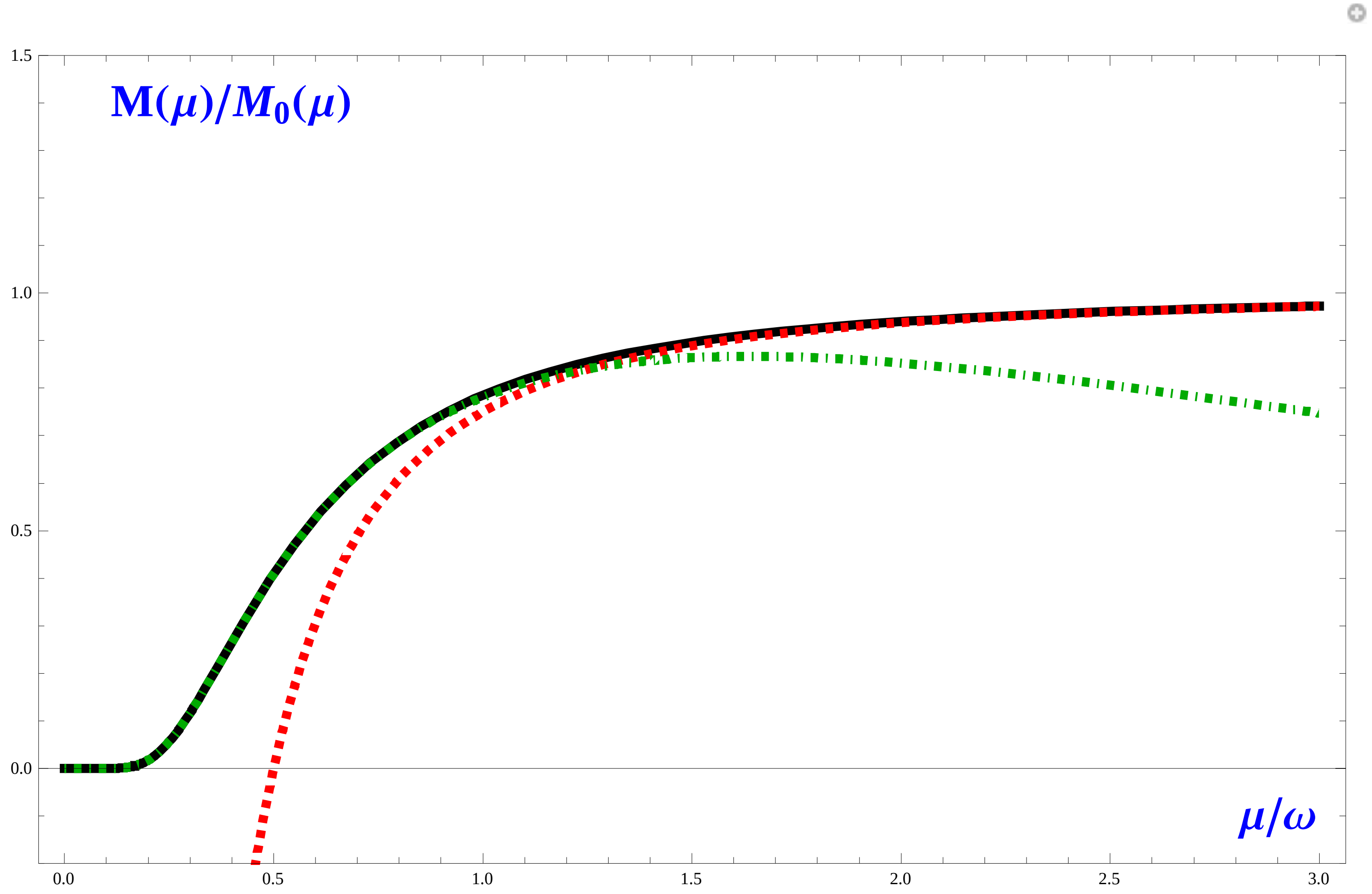
d

nPert

nRes

Exact	
OPE	
Reson.	

D	N_{pert}	N_{res}
3	2	3

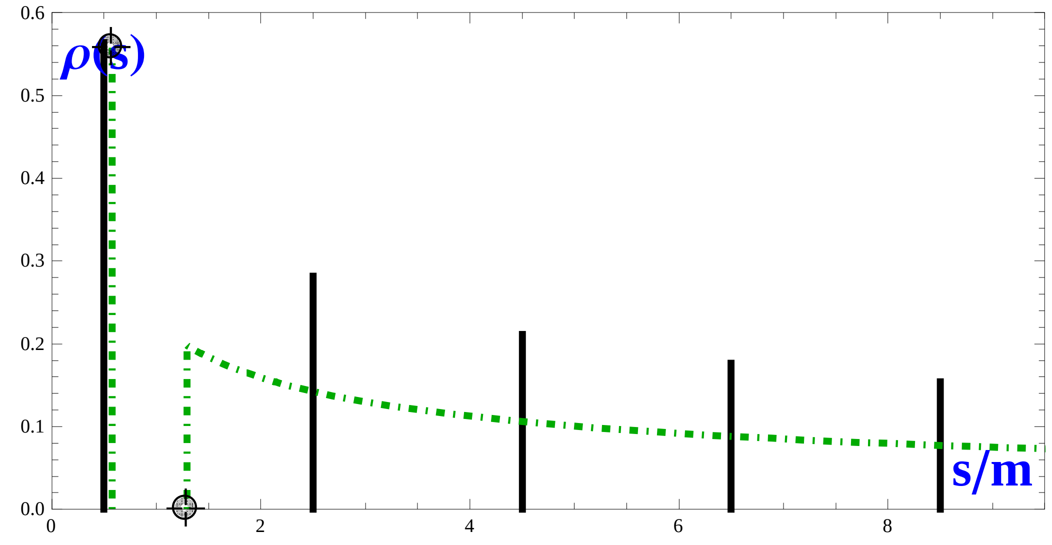


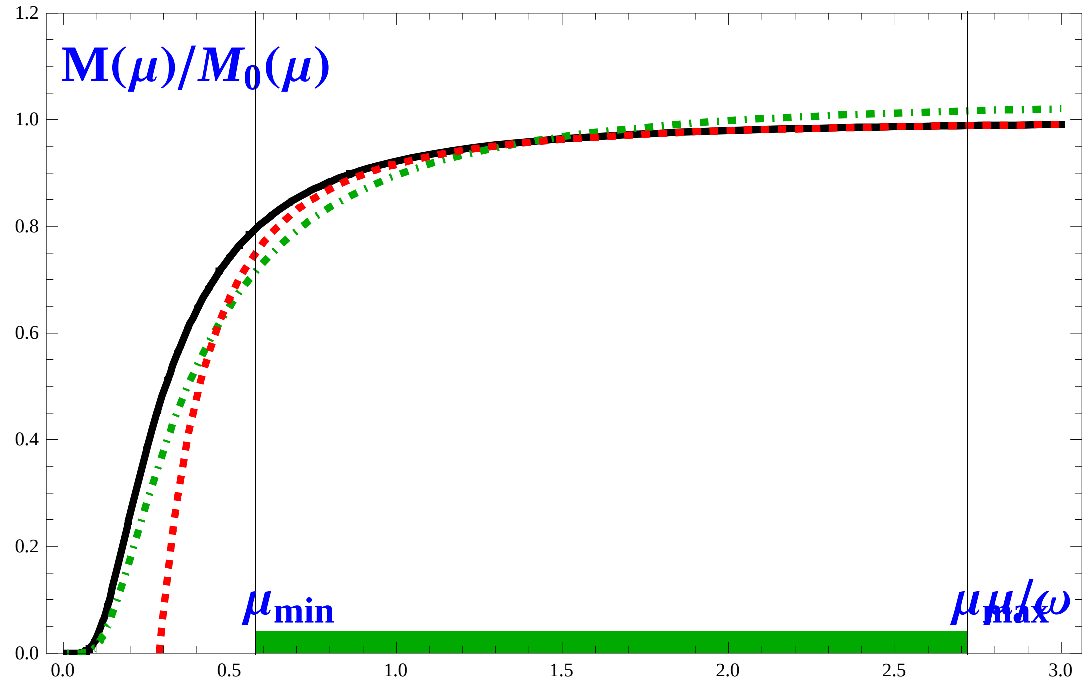
- Resonance correlator model $\quad M^{reson}(\mu) \approx \sum_{k=0}^{N_{res}-1} |\Psi_k(0)|^2 e^{-E_k/\mu}$
- OPE correlator model $\quad M^{OPE}(\mu) \approx \sum_{k=0}^{N_{res}} C_k \left(\frac{\omega}{\mu}\right)^{2k}$

Fit of parameters in SR

D

$\mathbf{n_{Pert}}$





D	N_{pert}
1	2

Exact	———
OPE	------
1+continuum	-·-·-·-

	Ψ_0^2	E_0	s_0
Exa⋱ ct	0.5⋱ 6⋱ 4	0.5	1.5
Here	0.5⋱ 5⋱ 8	0.5⋱ 7⋱ 4	1.2⋱ 9⋱ 2

μ_{min}	μ_{max}
0.58	2.72

We use here $m = 1$, $\omega = 1$.

- Contimuum term and highest order OPE term should contribute less than 33%.
- Window for Borel parameter $\mu_{min} < \mu < \mu_{max}$.

QCD Applications

- Masses
  - quarks: u, d, s, c, b
  - mesons: $\pi, \rho, J/\psi$, B, D,
  - baryons: N, Σ_*, Δ, Ξ_b, Ξ_c, Ω_b
- Decay constants: π, ρ, $D \to K, K^*, \pi, \rho$, $B \to K^*, K, D^*, \rho, \pi$
- Form factors: $\pi \to \pi\gamma^*$, $\gamma\gamma^* \to \pi$
- Distribution Amplitudes: π, ρ, a_0, f_0, K_0^*
- Valence quark distribution of π meson
- $\sigma_{e^+ e^-} \to$ **hadrons**, τ decay
- Tetraquark states $D_{sJ}^+(2317)$, $\chi(3872)$
- Exotic pentaquark state Θ^+

QCD condensate determination

Table 4: Values of the QCD condensates from QSSR.

Condensates	Values $[\mathrm{GeV}]^d$	Sources
$\langle\bar{u}u\rangle(2)$	$-(0.254 \pm .015)^3$	(pseudo)scal,
$\langle\bar{d}d\rangle/\langle\bar{u}u\rangle$	$1-9\times 10^{-3}$	non-norm. ord. (pseudo)scal,
$\langle\bar{s}s\rangle/\langle\bar{d}d\rangle$	0.74(3)	non-norm. ord. (pseudo)scal
		$\oplus$ light & heavy baryons
$\langle\alpha_s G^2\rangle$	$7(2)10^{-2}$	e^+e^-, $\Upsilon - \eta_b$, J/ψ Laplace
		τ, J/ψ mom : unconclusive
$g\langle\bar{\psi}G\psi\rangle$	$M_0^2 = 0.80(2)$	Light baryons, B, B^*
$g^3\langle G^3\rangle$	$(31\pm 13)\langle\alpha_s G^2\rangle$	J/ψ-mom
$\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$	$(4.5\pm 0.3)10^{-4}$	$\Longrightarrow \rho = 2.1 \pm 0.2$

Gluon condensate $<\alpha_s G^2>$ was first extracted from J/ψ, and then applied to η_c within QCD SR. The QCD SR prediction of η_c mass was $m_{\eta_c}^{\mathrm{QCD\,SR}} = 3.00 \pm 0.03\,\mathrm{GeV}$.
Latter η_c state was discovered with mass $m_{\eta_c}^{\exp} = 2.9803 \pm 0.0012\,\mathrm{GeV}$.

Conclusion

- Sum Rules as a virtual laboratory for QCD
- From Experiment to nonperturbative QCD vacuum properties
- From OPE to physical observables