Backup slides

Coherence production conditions

Coherence production conditions:

$$|\Delta E| \ll \sigma_E\,, \qquad\qquad |\Delta p| \ll \sigma_p\,.$$

On the other hand:

$$\Delta E \simeq v_g \Delta p + \frac{\Delta m^2}{2E}.$$

Constraint $|\Delta E| \ll \sigma_E \quad \Rightarrow$

$$\left|\frac{v_g \Delta p}{\sigma_E} + \frac{\Delta m^2}{2E\sigma_E}\right| \ll 1. \qquad\qquad (*)$$

(a) The two terms in ΔE do not approximately cancel each other. $\Rightarrow$ $v_g|\Delta p| \ll \sigma_E \le \sigma_p$, i.e. for relativistic neutrinos $|\Delta p| \ll \sigma_p$ follows from $|\Delta E| \ll \sigma_E$.

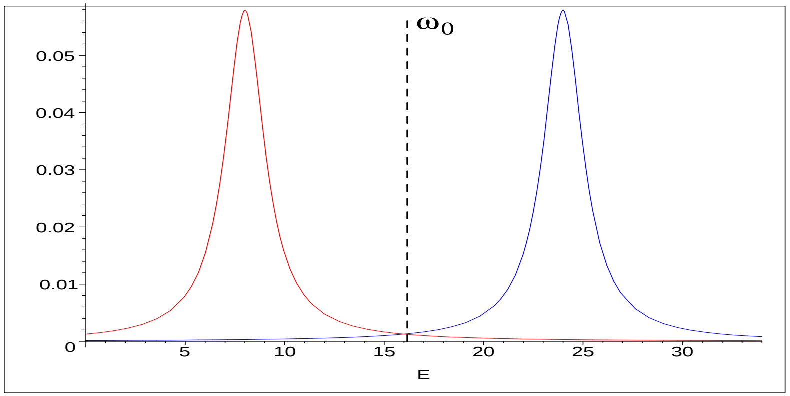
(b1) There is a strong cancellation, but both terms on the l.h.s. of (*) are small – see case (a).

(b2) Strong cancellation, but both terms on the l.h.s. of (*) are $\gtrsim 1$: momentum condition is independent. But: the only known case – Mössbauer neutrinos.

Mössbauer effect

Conventional Mössbauer effect – Res. absorption of γ quanta:

$$A^* \rightarrow A + \gamma; \qquad A + \gamma \rightarrow A^*$$

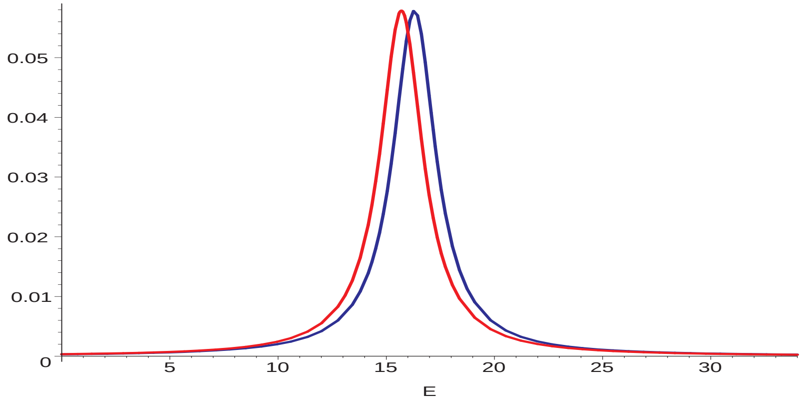


Nuclear exc. energy: ω_0.

Recoil energy: $R = \frac{\omega_0^2}{2M}$

$$E_e = \omega_0 - \frac{\omega_0^2}{2M}$$

$$E_a = \omega_0 + \frac{\omega_0^2}{2M}$$



Recoilless emission and absorption (Mössb. eff.):

$$E_e \simeq E_a \simeq \omega_0$$

Strong enhancement of absorption

Mössbauer effect with neutrinos?

Beta decay with 2 - body final state:

$$A(N,Z) \to A(N-1,Z+1) \; + e_B^- \; + \; \bar{\nu}_e$$

Inverse process:

$$\bar{\nu}_e \; + \; e_B^- \; + \; A(N-1,Z+1) \to A(N,Z)$$

If the neuclei are embedded in solid state lattice, recoilless emission and absorption in principle possible.

Possibility of Mössbauer effect with neutrinos:

Visscher, 1959; Kells & Schiffer, 1983; Raghavan, 2005, 2006

Relevant processes considered:

Bahcall, 1961 – bound state β decay;

Mikaelyan, Tsinoev & Borovoi, 1967 – inverse process (stimulated K-electron capture)

Mössbauer effect with neutrinos?

Mössbauer effect with neutrinos on ^{3}H – ^{3}He system:

$$^3\mathrm{H} \to (^3\mathrm{He} + e^-_B) + \bar{\nu}_e\,; \qquad \bar{\nu}_e + (^3\mathrm{He} + e^-_B) \to {}^3\mathrm{H}$$

Energy release: $Q = 18.6$ keV. Mean lifetime of ^{3}H is 17.8 yr $\Rightarrow$
Nat. linewidth $\Gamma_{^3\mathrm{H}} = 1.17 \times 10^{-24}$ eV – extremely small: $\Delta E/E \sim 10^{-28}$!

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Various (homogeneous and inhomogeneous) broadening effects exist. By suppressing them probably an effective linewidth $\Gamma_{\mathrm{eff}} \sim 10^{-11}\ \mathrm{eV}$ can be achieved (W. Potzel) $\Rightarrow$ $\Delta E/E \sim 10^{-15}$ – still very small.

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Number of ^{3}H atoms produced in the target can be counted by detecting their decay or using mass spectroscopy.

Very serious technical difficulties exist, but apparently realization of a Mössbauer experiment with neutrinos is not impossible (Raghavan, Potzel).
If realized: for $\Gamma \sim 10^{-11}$ eV, $\sigma \sim 10^{-33}\ cm^2$!

Mössbauer effect with neutrinos?

If a Mössbauer neutrino experiment is realized $\Rightarrow$ a unique source of extremely monochromatic low energy neutrinos. Would open up possibilities

- to detect for the first time keV neutrinos
- to detect neutrinos with g or 100 g scale (rather than t or kt scale) detectors
- to observe gravitational redshift of neutrinos
- to study neutrino oscillations at distances ~ 10 m rather than km or hundreds/thousands of km
- to search for the effects of yet unmeasured mixing angle θ_{13} and possibly measure it
- to discriminate between the normal and inverted neutrino mass hierarchies without using matter effects
- to study possible oscillations into sterile neutrino states

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Arguments in the literature (Bilenky et al.):

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Is that true?

Will Mössbauer neutrinos oscillate?

Neutrino oscillations require some intrinsic uncertainty of energy and momentum of the emitted and detected neutrino states !

If E and p were known precisely, from $E^2 = p^2 + m_i^2$ one would determine which mass eihenstate has been emitted $\Rightarrow$ neutrinos of different mass would not be emitted coherently.

For Mössbauer effect with neutrinos in ^{3}H – ^{3}He system:

$$\diamondsuit \quad \frac{\Delta m^2}{2E} = \frac{2.5\times 10^{-3}\,\mathrm{eV}^2}{2\cdot 18.6\,\mathrm{keV}} \simeq 6.7\cdot 10^{-8}\,\mathrm{eV} \gg \Gamma \sim 10^{-11}\,\mathrm{eV}\,!$$

Can neutrinos of different mass be accommodated within such a small energy uncertainty?

Will neutrinos with such small energy uncertainty oscillate ?

Two “standard” approaches to ν oscillations

The oscillation phase: $\phi = p_\mu x^\mu = E\cdot t - p\cdot x \quad \Rightarrow$

$$\Delta\phi \;=\; \Delta E\!\cdot\! t \;-\; \Delta p\!\cdot\! L$$

I. Same momentum approach $(\Delta p \;=\; 0)$. The oscillation phase

$$\Delta\phi \;=\; \Delta E\cdot t \;-\; \Delta p\cdot L \;\;\Rightarrow\;\; \Delta E\cdot t$$

– evolution in time; needs to use $L \;\simeq\; t$.

II. Same energy approach $(\Delta E \;=\; 0)$:

$$\Delta\phi \;=\; -\,\Delta p\cdot L$$

– evolution in space.

Will Mössbauer neutrinos oscillate?

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Our point of view: in general, there is no reason to believe that ν_i have either same energy or same momentum. No need to perform Mössbauer ν experiment to decide which approach is correct – it is sufficent to carefully examine the validity of the approximations used.

How about Mössbauer neutrinos?

Very small effective linewidth Γ $\Rightarrow$ small energy uncertainty of the emitted neutrino state. Can different neutrino mass eigenstates be emitted coherently?

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2\right]^{1/2}$$

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$\Rightarrow \quad \sigma_p \sim 10\ {\rm keV}$, i.e. $\sigma_m^2 \simeq 2p\sigma_p \sim 4\times 10^8\ {\rm eV}^2 \gg \Delta m^2$

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$\Rightarrow$ $\sigma_p \sim 10\ {\rm keV}$, i.e. $\sigma_m^2 \simeq 2p\sigma_p \sim 4\times 10^8\ {\rm eV}^2 \gg \Delta m^2$

$\Rightarrow$ Oscillations must occur !

QFT calculation

Inhomogeneous line broadening: Calculate the probability of the overall process for zero linewidths and then average the result over the energy distribution of ^{3}H and ^{3}He nuclei in the source and detector.

Homogeneous line broadening: modify the amplitude of the process and apply a proper averaging procedure to take into account the stochastic nature of the processes leading to homog. broadening. $\Rightarrow$ Results in both cases are formally very similar. Mössbauer res. condition:

$$|E_S - E_D| \ll \gamma_S + \gamma_D$$

If it is satisfied $\Rightarrow$ neutrino detection cross section enhanced by a factor

$$\sim (\alpha Z m_e)^3/[p_e E_e(\gamma_S + \gamma_D)] \sim 10^{12}$$

compared to non-resonance $\sigma(\bar{\nu}_e + A \to A' + e^+)$ for neutrinos of same energy (assuming recoil-free fraction ~ 1).

QFT calculation – contd.

The amplitude for zero linewidths:

$$i\mathcal{A} \;=\; \int d^3x_1\, dt_1 \int d^3x_2\, dt_2\, \Psi^*_{He,S}(\vec{x}_1)e^{+iE_{He,S}\,t_1}\;\Psi_{H,S}(\vec{x}_1)e^{-iE_{H,S}\,t_1}$$

$$\cdot\;\Psi^*_{H,D}(\vec{x}_2)e^{+iE_{H,D}\,t_2}\;\Psi_{He,S}(\vec{x}_2)e^{-iE_{He,D}\,t_2}$$

$$\cdot\sum_j \mathcal{M}^\mu_S \mathcal{M}^{\nu *}_D |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)}$$

$$\cdot\;\bar{u}_{e,S}\gamma_\mu(1-\gamma_5)\,\frac{i(\not{p}+m_j)}{p_0^2-\vec{p}^2-m_j^2+i\epsilon}\,\gamma_\nu(1-\gamma_5)u_{e,D}$$

Here

$$\mathcal{M}^\mu_{S,D} = \frac{G_F\cos\theta_c}{\sqrt{2}}\,\psi_e(R)\,\bar{u}_{He}\,(M_V\,\delta^\mu_0 - g_A M_A\sigma_i\,\delta^\mu_i/\sqrt{3})\,u_H\,\kappa^{1/2}_{S,D}$$

QFT calculation – contd.

The overall process rate:

$$\Gamma = \frac{\Gamma_0 \, B_0}{4\pi L^2} \, Y_S Y_D \int_0^\infty dE_{H,S} \, dE_{He,S} \, dE_{He,D} \, dE_{H,D}$$

$$\cdot \, \delta(E_S - E_D) \rho_{H,S}(E_{H,S}) \, \rho_{He,D}(E_{He,D}) \, \rho_{He,S}(E_{He,S}) \, \rho_{H,D}(E_{H,D})$$

$$\cdot \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \, \exp\left[- \frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2} \right] e^{i\left(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2}\right)L}$$

σ_p – effective momentum uncertainty of the emission/absorption processes:

$$\frac{1}{\sigma_p^2} = \frac{1}{m_H \omega_{H,S} + m_{He} \omega_{He,S}} + \frac{1}{m_H \omega_{H,D} + m_{He} \omega_{He,D}} \, ,$$

An analogue of the Debye - Waller (Lamb - Mössbauer) factor:

$\diamondsuit$ $$\exp[-(2E_S^2 - m_j^2 - m_k^2)/2\sigma_p^2] \; = \; \exp[-(p_j^2 + p_k^2)/2\sigma_p^2]$$

QFT calculation – contd.

For Lorentzian energy distributions of external particles:

$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B}-E_{A,B,0})^2+\gamma_{A,B}^2/4}$$

$(A=\{H,He\},\ B=\{S,D\},\ E_{A,B,0}=m_A+\frac{1}{2}\omega_{A,B})\quad\Rightarrow$

$$\Gamma \simeq \frac{\Gamma_0\,B_0}{4\pi L^2}\,Y_S Y_D \sum_{j,k}|U_{ej}|^2|U_{ek}|^2 \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right]\exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

$$\cdot\,\frac{1}{2}\left(e^{-L/L_{jk,S}^{\rm coh}}+e^{-L/L_{jk,D}^{\rm coh}}\right)\exp\left[-i\frac{\Delta m_{jk}^2}{2\bar{E}}\,L\right]\frac{(\gamma_S+\gamma_D)/2\pi}{(E_{S,0}-E_{D,0})^2+\frac{(\gamma_S+\gamma_D)^2}{4}}$$

$L_{jk,B}^{\rm coh}$ – coherence lengths:

$$L_{jk,B}^{\rm coh} = \frac{4\bar{E}^2}{\gamma_B|\Delta m_{jk}^2|} = \frac{\sigma_x}{\Delta v_g}\,, \qquad \sigma_x = \frac{2}{\gamma_B} \qquad (B=S,D)$$

QFT calculation – contd.

Generalized Lamb – Mössbauer (Debye – Waller) factor

$$\exp\left[-\frac{p_j^2+p_k^2}{2\sigma_p^2}\right] = \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right]\exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

First factor $\Rightarrow$ suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor $\Rightarrow$ suppression of oscillations.

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$|\Delta m_{jk}^2| \lesssim 2\sigma_p^2 \quad \Rightarrow$ localization condition: Spatial localization $\sigma_x \sim 1/\sigma_p$.
Oscillations would be suppressed only if $|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2$.

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In reality: $|\Delta m_{jk}^2|_{\max} \simeq 2.5\cdot 10^{-3}\ \mathrm{eV}^2$; $\sigma_p^2 \sim (10\ \mathrm{keV})^2$ $\Rightarrow$
oscillations will not be suppressed.

QFT calculation – contd.

For realistic values of parameters – just the expected result: the rate of no-oscillation production-detection process times the standard oscillation probability (probability of $\bar{\nu}_e$ survival). Decoherence and delocalization can be neglected.

Conclusion:

If a Mössbauer neutrino experiment is realized – recoillessly emitted and absorbed neutrinos will oscillate.

Longitudinal *vs.* transversal w.p. dispersion

Spreading of the wave packets: consequence of the fact that the there is a spread of momenta inside of the wave packets and of the p-dependence of the group velocity.

$$v^i_{spr} \simeq \frac{\partial v_i}{\partial p^j}\sigma_p^j = \frac{1}{E}(\delta_{ij} - v_i v_j)\sigma_p^j = \frac{1}{E}[\sigma_p^i - v_i(\vec{v}\vec{\sigma_p})]$$

This gives

$$v^{\perp}_{spr.} = \frac{\sigma_p}{E}\,, \qquad v^{||}_{spr.} = \frac{\sigma_p}{E}(1-v^2) = \frac{\sigma_p}{E}\frac{m^2}{E^2}$$

$t_{transv} \sim E/\sigma_p^2, \;\; t_{long.} \sim E^3/\sigma_p^2 m^2$.