Axial anomaly, quark-hadron duality and transition form factors of pseudoscalar mesons

Yaroslav Klopot[1], Armen Oganesian[1,2], Oleg Teryaev[1]

[1]Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia

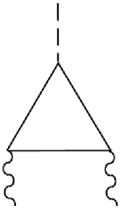
[2]Institute of Theoretical and Experimental Physics, Moscow, Russia

Based on: ArXiv:1106.3855[hep-ph]

Baikal Summer School on Physics of Elementary Particles and Astrophysics
Bolshie Koty, Russia

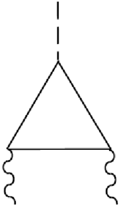
3 – 10 July 2011

Outline



- The phenomenon of axial anomaly is known to be one of the most subtle effects of quantum field theory. It is well known, that axial anomaly governs two-photon decays of pseudoscalar mesons.
- It is often assumed that axial anomaly is manifested in real photon amplitudes only. This is not true – it appears at any Q^2!
- By means of the dispersive approach to axial anomaly it is possible to derive anomaly sum rule (ASR) providing a handy tool to study the meson spectrum even beyond the factorization hypothesis. Being an exact tool, the ASR allows to get a relation between the corrections to continuum and to lower lying states.

Anomaly sum rule



The VVA triangle graph correlator

$$T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0 | T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\} | 0 \rangle \qquad (1)$$

contains axial current $J^{(8)}_{\alpha 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s)$ and two vector currents $J_\mu = (e_u \bar{u}\gamma_\mu u + e_d \bar{d}\gamma_\mu d + e_s \bar{s}\gamma_\mu s)$; k, q are momenta of photons.

$T_{\alpha\mu\nu}(k,q)$ can be presented as a tensor decomposition:

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) &= F_1\,\varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\,\varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+ F_3\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\ &+ F_5\,k_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\,q_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma \end{aligned} \tag{2}$$

We are interested in the following case: $k^2 = 0, Q^2 = -q^2 > 0$.
Dispersive approach to axial anomaly leads to [Hořejší&Teryaev'95]:

$$\int_{4m^2}^{\infty} A_3(s; Q^2, m^2)ds = \frac{1}{2\pi}N_c C^{(8)}, \tag{3}$$

$$A_3 \equiv \frac{1}{2}Im(F_3 - F_6), \quad C^{(8)} \equiv \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}$$

- Holds for any Q^2 and any m^2.
- Does not have α_s corrections (Adler-Bardeen theorem).
- Does not have non-perturbative corrections (t'Hooft consistency principle).

Mesons contributions to ASR

Saturating the l.h.s. of the 3-point correlation function (1) with the resonances and singling out their contributions to ASR (27) we get the sum of resonances with appropriate quantum numbers:

$$f^8_\eta F_\eta + f^8_{\eta'} F_{\eta'} + (other\ resonanses) = \int_{4m^2}^{\infty} A_{3a}(t; q^2, m^2) dt = \frac{1}{2\pi} N_c C^{(8)}. \tag{4}$$

Here the projections of the axial current $J^{(a)}_{5\alpha}$ onto one-meson states $M(= \eta, \eta')$ define the coupling (decay) constants f^a_M:

$$\langle 0 | J^a_{\alpha 5}(0) | M(p) \rangle = i p_\alpha f^a_M \ . \tag{5}$$

The form factors $F_{M\gamma}$ of the transition $\gamma\gamma^* \to M$ are defined by the matrix elements:

$$\int d^4x e^{ikx} \langle M(p) | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \ . \tag{6}$$

- The relation (4) is exact and expresses the global duality between hadrons and quarks. Nevertheless, to analyze the hadron properties one should additionally implement the local quark-hadron duality hypothesis. Taking into account large $\eta - \eta'$ mixing, one can express the spectral function $A_{3a}(s, Q^2)$ in a form of "two resonanses+continuum":

$$A_{3a}\left(s, Q^2\right) = \pi f_\eta^8 \delta(s - m_\eta^2) F_{\eta\gamma}\left(Q^2\right) + \pi f_{\eta'}^8 \delta(s - m_{\eta'}^2) F_{\eta'\gamma}\left(Q^2\right)$$

$$+ A_{3a}^{QCD} \theta(s - s_0).$$

Here s_0 is a continuum threshold and A_{3a}^{QCD} at one-loop level is

$$A_{3a}^{QCD} = \frac{1}{2\pi\sqrt{6}} \frac{Q^2}{(s + Q^2)^2}. \qquad (7)$$

- There is no two-loop α_s corrections to this expression [Jegerlehner,Tarasov'05]

- The particles with non-zero two-photon decays cannot be included to continuum as it vanishes at $Q^2 = 0$, so they should be taken into account explicitly in the ASR.
- For heavy mesons the corresponding coupling constants should be suppressed at least as $(m_\eta/m_{res})^2$ which follows from the conservation of axial current J_8 in the chiral limit (if only strong interaction is taken into account).
- That is why we restrict ourself only to η and η' mesons.

  The ASR for the *octet* channel then reads:

$$\pi f_\eta^8 F_{\eta\gamma}(Q^2) + \pi f_{\eta'}^8 F_{\eta'\gamma}(Q^2) = \frac{1}{2\pi\sqrt{6}} \frac{s_0}{Q^2 + s_0}. \qquad (8)$$

- This relation is valid for all Q^2 due to the absence of the corrections which allows to utilize it for different Q^2.

ASR at $Q^2 = 0$: real photons

$$\pi f_\eta^8 F_{\eta\gamma}(0) + \pi f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\pi\sqrt{6}} \tag{9}$$

where

$$F_{M\gamma}(0) = \widetilde{A}_{M\to 2\gamma} = \sqrt{\frac{4\Gamma_{M\to 2\gamma}}{\pi\alpha^2 m_M^3}}, \tag{10}$$

where $\Gamma_{M\to 2\gamma}$ and m_M are two-photon decay widths and masses of η, η' mesons.

ASR in the asymptotics: $Q^2 \to \infty$

The asymptotics of transition form factors at large Q^2 (there are contributions from both 8th and 0th components of axial current):

$$\frac{F^{as}_{\eta\gamma}}{F^{as}_{\pi\gamma}} = \frac{f^8_\eta \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) + f^0_\eta \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2)}{f_\pi \frac{1}{\sqrt{2}}(e_u^2 - e_d^2)} = \frac{f^8_\eta}{f_\pi}\frac{1}{\sqrt{3}} + \frac{f^0_\eta}{f_\pi}\frac{2\sqrt{2}}{\sqrt{3}},$$

$$\frac{F^{as}_{\eta'\gamma}}{F^{as}_{\pi\gamma}} = \frac{f^8_{\eta'} \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) + f^0_{\eta'} \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2)}{f_\pi \frac{1}{\sqrt{2}}(e_u^2 - e_d^2)} = \frac{f^8_{\eta'}}{f_\pi}\frac{1}{\sqrt{3}} + \frac{f^0_{\eta'}}{f_\pi}\frac{2\sqrt{2}}{\sqrt{3}},$$

$$F^{as}_{\pi\gamma} = \frac{\sqrt{2}f_\pi}{Q^2}.$$

Then the ASR for the octet channel takes the form:

$$\pi f^8_\eta \left(f^8_\eta \frac{\sqrt{2}}{\sqrt{3}} + f^0_\eta \frac{4}{\sqrt{3}}\right) + \pi f^8_{\eta'}\left(f^8_{\eta'}\frac{\sqrt{2}}{\sqrt{3}} + f^0_{\eta'}\frac{4}{\sqrt{3}}\right) = \frac{s_0}{2\pi\sqrt{6}}, \qquad (11)$$

ASR for $\eta - \eta'$ system vs. BaBar

- It is convenient to divide both sides of ASR by $f_8 = \sqrt{(f_\eta^8)^2 + (f_{\eta'}^8)^2}$. Consider the form factors multiplied by Q^2:

$$\frac{Q^2}{f_8}(f_\eta^8 F_{\eta\gamma}(Q^2) + f_{\eta'}^8 F_{\eta'\gamma}(Q^2)) =$$

$$\frac{\frac{Q^2}{f_8}\sqrt{\frac{2}{3}}}{4\pi^2 + Q^2/((f_\eta^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0])}.$$

η and η'

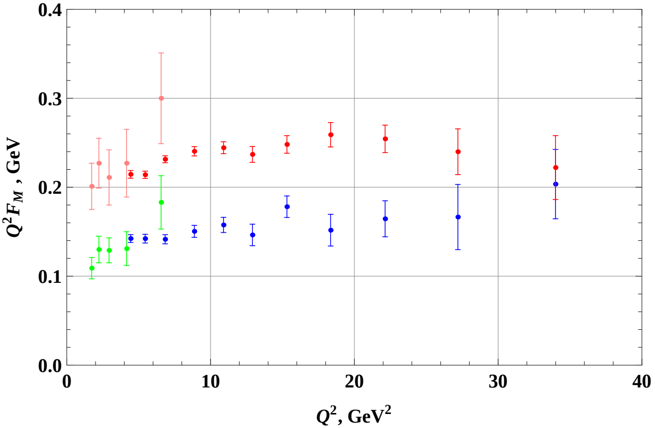


Figure: Experimental data on transition form factors: η (CLEO-green, BABAR-blue), η' (CLEO-pink, BABAR-red)[BaBar Collaboration: PoS (ICHEP 2010) **144**, arXiv:1101.1142]

One-angle mixing scheme:

$$\mathbf{F} \equiv \begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta & f_8 \sin\theta \\ -f_0 \sin\theta & f_0 \cos\theta \end{pmatrix}. \quad (12)$$

$$Q^2 (F_{\eta\gamma}(Q^2)\cos\theta + F_{\eta'\gamma}(Q^2)\sin\theta) = \sqrt{\frac{2}{3}} \frac{Q^2}{4\pi^2 f_8 + Q^2/f_8}, \quad (13)$$

$$f_8 = \frac{\alpha}{4\sqrt{6}\pi^{3/2}} \left(\sqrt{\frac{\Gamma_{\eta\to 2\gamma}}{m_\eta^3}} \cos\theta + \sqrt{\frac{\Gamma_{\eta'\to 2\gamma}}{m_{\eta'}^3}} \sin\theta \right)^{-1}. \quad (14)$$

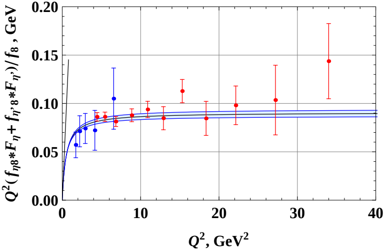


Figure: ASR for one-angle mixing scheme : $\theta = -14^o$

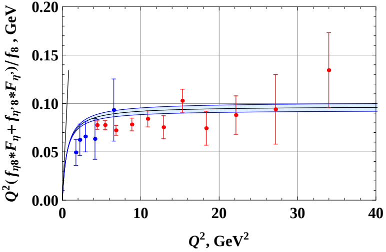


Figure: ASR for one-angle mixing scheme: $\theta = -16^o$

Schemes with 2 mixing angles (Leutwyler'97; Feldmann,Kroll,Stech'98; Escribano'05):

$$\mathbf{F} \equiv \begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 & 0 \\ 0 & f_0 \end{pmatrix} \begin{pmatrix} c_8 & s_8 \\ -s_0 & c_0 \end{pmatrix}, c_i \equiv \cos\theta_i, s_i \equiv \sin\theta_i. \quad (15)$$

Parameters:

Feldmann,Kroll,Stech'98:

$f_0 = 1.17 f_\pi, f_8 = 1.26 f_\pi, \theta_0 = -9.2^o, \theta_8 = -21.2^o,$

Escribano'05: $f_0 = 1.29 f_\pi, f_8 = 1.51 f_\pi, \theta_0 = -2.4^o, \theta_8 = -23.8^o.$

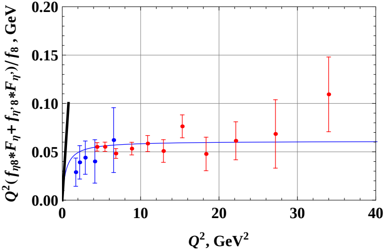


Figure: ASR for scheme Feldmann,Kroll,Stech'98: $f_0 = 1.17 f_\pi, f_8 = 1.26 f_\pi, \theta_0 = -9.2^o, \theta_8 = -21.2^o$

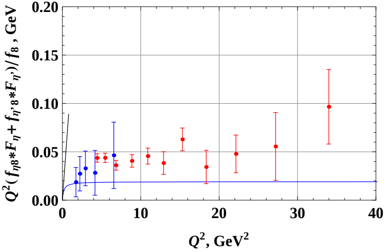


Figure: ASR for scheme Escribano'05: $f_0 = 1.29 f_\pi, f_8 = 1.51 f_\pi, \theta_0 = -2.4^o, \theta_8 = -23.8^o$

Summary

- We generalize the rigorous non-perturbative QCD approach which does not contain any free adjustable parameters to the case of η and η' mesons where mixing is crucially important.
- Combining the exact dispersive form of anomaly relation, quark-hadron duality hypothesis and asymptotic matching with QCD factorization we express the combination of η, η' meson transition form factors in terms of meson decay constants only. The obtained anomaly sum rule is valid in the whole kinematical region starting from $Q^2 = 0$.
- Our analysis shows that for a large number of mixing schemes ASR is in a good agreement with experimental data (probably with the exception of parameters offered in Escribano'05).
- ASR requires unexpectedly small interval of duality for the $\eta - \eta'$ system ($s_0 \lesssim 1 GeV^2$)– large corrections effectively increasing s_0 are possible.

Thank you for your attention!

Backup

Bose symmetry implies:

$$\begin{aligned} F_1(k,p) &= -F_2(p,k), \\ F_3(k,p) &= -F_6(p,k), \\ F_4(k,p) &= -F_5(p,k). \end{aligned} \tag{16}$$

One can show also that

$$F_6(k,p) = -F_3(k,p) \tag{17}$$

vector Ward identities

$$k^\mu T_{\alpha\mu\nu} = 0, \qquad p^\nu T_{\alpha\mu\nu} = 0 \tag{18}$$

In terms of formfactors, the identities (18) read

$$\begin{aligned} F_1 &= k.p\, F_3 + p^2\, F_4 \\ F_2 &= k^2\, F_5 + k.p\, F_6 \end{aligned} \tag{19}$$

Anomalous axial-vector Ward identity for the amplitude (2) is [Adler'69]:

$$q^\alpha T_{\alpha\mu\nu}(k,p) = 2m T_{\mu\nu}(k,p) + \frac{1}{2\pi^2}\varepsilon_{\mu\nu\rho\sigma} k^\rho p^\sigma \tag{20}$$

Backup

$$T_{\mu\nu}(k,p) = G\ \varepsilon_{\mu\nu\rho\sigma} k^\rho p^\sigma \tag{21}$$

where G is the relevant form factor. In terms of form factors, eq.(20) reads

$$F_2 - F_1 = 2mG + \frac{1}{2\pi^2} \tag{22}$$

For the form factors F_3, F_4 and G one may write unsubtracted dispersion relations

$$F_j(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t-q^2}dt, \qquad j = 3,4 \tag{23}$$

$$G(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{B(t)}{t-q^2}dt$$

Backup

From (17) and (19) it is easy to see that for the considered kinematical configuration one has

$$F_2 - F_1 = (p^2 - q^2)F_3 - p^2 F_4 \tag{24}$$

Using now (23) and taking into account that the imaginary parts of the relevant formfactors satisfy non-anomalous Ward identities, in particular

$$(p^2 - t)A_3(t) - p^2 A_4(t) = 2mB(t) \tag{25}$$

one gets finally

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t)dt \tag{26}$$

Comparing eq.(26) with (22) one may thus observe that the occurrence of the axial anomaly is equivalent to a "sum rule"

$$\int_{4m^2}^{\infty} A_3(t; p^2, m^2)dt = \frac{1}{2\pi} \tag{27}$$

(which must hold for an arbitrary m and for any p^2 in the considered region).