

# Higgs models: part 2

Igor Ivanov

University of Liège, Belgium,  
and  
Institute of Mathematics, Novosibirsk

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- 1 SM+ 4th fermion generation
- 2 Two Higgs doublets
- 3 More Higgses in various representations
- 4 Further reading

# Some terminology

The Standard Model contains: (1) **gauge interactions**; (2) **matter fields**; (3) **Higgs mechanism**.

- If all sectors are minimal, we call it the **minimal Standard Model**.
- If the gauge and Higgs sectors are minimal, but we assume that there exist another generation of fundamental fermions, we call it **SM with 4th generation**.

NB: the masses of the 4th generation fermions arise from the same standard Higgs mechanism.

This is a small step (almost) beyond the SM.

## 4th fermion generation

No  $q'$  found at colliders:  $m_{q'} > 200\text{--}250$  GeV. Precise measurements of the Z-shape at LEP ruled out **light** 4th generation neutrinos. If 4th generation exists, its fermions must be heavy.

Is it possible to observe their effects without directly producing them?

Yes, through their strong effect on the effective coupling of the Higgs to gluons  $Hgg$ .

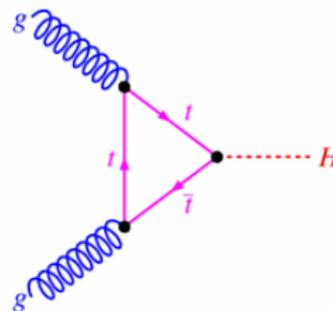
# $Hgg$ coupling

Gluons are massless  $\rightarrow$  **no tree-level  $Hgg$  vertex.**

Effective coupling via loops:

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{8\pi v} G_{\mu\nu}^a G^{a\mu\nu} H \cdot \sum_q F(x_q).$$

where  $F(x_q)$  is a certain function depending on  $x_q = m_q^2/m_H^2$ .  $F(x)$  is small at small  $x$  and approaches  $-4/3$  at  $x \gg 1$ .



Although a very heavy quark loop is suppressed by  $M_q$ , this suppression is compensated by the  $Hq\bar{q}$  coupling  $M_q/v$ .

# $Hgg$ coupling

In the minimal SM the  $Hgg$  coupling is dominated by the  $t$ -loop.  
 If 4th generation of heavy quarks exists,  $Hgg$  coupling is  $\approx 3$  stronger.

Phenomenological consequence:  $\sigma(pp \rightarrow H)$  increases by an order of magnitude.

The effect is so dramatic that it can be directly probed at the Tevatron and the LHC.

Current results: the 4th generation is incompatible with the SM Higgs with mass  $M_H = 131 - 204$  GeV (Tevatron) and  $M_H = 144 - 207$  GeV (LHC).

# Two doublets

Two-Higgs-doublet model suggested by T.D.Lee in 1973. [Two Higgs doublets](#)

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}.$$

interact with the gauge bosons,  $|D_\mu \phi_i|^2$ , with themselves via the Higgs potential constructed from  $(\phi_i^\dagger \phi_j)$ , and with fermions via Yukawa interactions.

Two main physics motivation of 2HDM:

- [MSSM](#) contains two Higgs doublets.
- a source of [CP-violation](#) from the Higgs sector.

**8 degrees of freedom**: 3 Goldstone bosons ( $\rightarrow W_L^\pm, Z_L$ ) + **5 physical Higgses** ( $H^\pm$ , three neutrals).

## EWSB in 2HDM

A simple yet illustrative version of 2HDM:

$$V = -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) + \lambda(\phi_1^\dagger\phi_1)^2 + \lambda(\phi_2^\dagger\phi_2)^2 \\ + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) - \frac{\lambda_5}{2} \left[ (\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2 \right].$$

Search for the minimum in the following form

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

Useful notation:  $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$ ,  $v_1 = v \cos \beta$ ,  $v_2 = v \sin \beta$ ,  
 $\tan \beta = v_2/v_1$ .

## EWSB in 2HDM

$$\frac{\partial V}{\partial v_1} = v_1 [-\mu_1^2 + \lambda v_1^2 + \lambda_{345} v_2^2], \quad \frac{\partial V}{\partial v_2} = v_2 [-\mu_2^2 + \lambda v_2^2 + \lambda_{345} v_1^2].$$

where  $\lambda_{34} \equiv \lambda_3 - \lambda_4 - \lambda_5$ . Second derivatives:

$$\frac{\partial^2 V}{\partial v_1^2} = [\dots]_1 + 2\lambda v_1^2, \quad \frac{\partial^2 V}{\partial v_2^2} = [\dots]_2 + 2\lambda v_2^2, \quad \frac{\partial^2 V}{\partial v_1 \partial v_2} = 2\lambda_{34} v_1 v_2.$$

Several possibilities exist depending on the parameters:

- Phase A:  $v_1 = v_2 = 0$  (EW symmetric).
- Phase B1:  $v_1 \neq 0, v_2 = 0$ ; Phase B2:  $v_2 \neq 0, v_1 = 0$ .
- Phase C:  $v_1, v_2 \neq 0$ .

Explicit conditions on  $\mu_1^2, \mu_2^2$  and  $\lambda$ 's can be easily found.

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## EWSB in 2HDM

Let's pick up the phase  $C$  ( $\tan \beta \neq 0, \infty$ ). Small fluctuations:

$$\phi_1 = \left( \frac{1}{\sqrt{2}}(v_1 + h_1 + i\eta_1) \right), \quad \phi_2 = \left( \frac{1}{\sqrt{2}}(v_2 + h_2 + i\eta_2) \right),$$

substitute in the potential and pick up quadratic terms:

$$\begin{aligned} V_2 &= (\lambda_4 + \lambda_5)(v_2 w_1^- - v_1 w_2^-)(v_2 w_1^+ - v_1 w_2^+) \\ &\quad + \lambda_5(v_2 \eta_1 - v_1 \eta_2)^2 + \lambda v_1^2 h_1^2 + \lambda v_2^2 h_2^2 + 2\lambda_{345} v_1 v_2 h_1 h_2. \\ &= m_{H^\pm}^2 H^- H^+ + \frac{1}{2} M_A^2 A^2 + \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_H^2 H^2, \end{aligned}$$

where

$$\begin{aligned} H^\pm &= \sin \beta w_1^\pm - \cos \beta w_2^\pm, & m_{H^\pm}^2 &= (\lambda_4 + \lambda_5)v^2, \\ A &= \sin \beta \eta_1 - \cos \beta \eta_2, & m_A^2 &= \lambda_5 v^2, \end{aligned}$$

while  $h$  and  $H$  are light and heavy linear combinations of  $h_1$  and  $h_2$ .

Goldstone fields  $G^\pm = \cos \beta w_1^\pm + \sin \beta w_2^\pm$  and  $G^0 = \cos \beta \eta_1 + \sin \beta \eta_2$  disappear.

# Fermion sector

There are several ways to **couple two Higgs doublets to fermions**:

- Type I: only  $\phi_2$  couples to all fermions;
- Type II:  $\phi_2$  couples to all up-type fermions,  $\phi_1$  couples to down-type fermions.
- Type X:  $\phi_2$  couples to quarks,  $\phi_1$  couples to leptons.

The constant in each Yukawa term is adjusted to give the right mass to the fermions, but the coupling to the Higgs bosons can be **enhanced or suppressed** with respect to SM: v.e.v.'s ( $v_1, v_2$ ) and the physical neutral Higgses ( $h, H$ ) are not aligned.

Phenomenological properties of all these Higgs bosons **can be very different from the SM**.

# Some phenomenological features

- The version we considered is **CP-conserving**: all neutral Higgses have definite *CP*-parity (*A* is *CP*-odd, *h* and *H* are *CP*-even). A more complicated Higgs potential can induce **CP-violation in the Higgs sector** (original motivation by T. D. Lee).
- **Decoupling limit**. It's possible that only *h* is sufficiently light,  $m_h \sim 100\text{--}200$  GeV, while all other Higgs bosons are very heavy,  $m_H, m_A, m_{H^\pm} \sim 1$  TeV. In this case *h* is usually **SM-like**. In this scenario it will be difficult to tell 2HDM from SM at the LHC.
- **Inert model**. If  $v_2 = 0$  and if  $\phi_2$  does not couple to fermions, the Higgs boson  $h_2$  becomes massive, stable and weakly interacting particle  $\rightarrow$  **dark matter candidate**.

# Charge-breaing vacuum

An exotic possibility in 2HDM:

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_2 \end{pmatrix}$$

with  $u \neq 0$ . The electric charge is not conserved anymore by the vacuum, the photon becomes massive  $\rightarrow$  **charge-violating vacuum**.

This is not the case in the present world, but **it might have taken place in the early Universe**. It is possible that when the early Universe was cooling down, it evolved through an intermediate stage of charge-breaking vacuum.

# General 2HDM potential

If we want to understand all the opportunities offered by two Higgs doublets, we need to consider the **most general renormalizable 2HDM potential**.

$$\begin{aligned}
 V = & -\frac{1}{2} \left[ m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2*}(\phi_2^\dagger\phi_1) \right] \\
 & + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
 & + \frac{1}{2} \left[ \lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2 \right] + \left\{ \left[ \lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2) \right] (\phi_1^\dagger\phi_2) + \text{h.c.} \right\}
 \end{aligned}$$

It contains **14 free parameters** and **it cannot be minimized explicitly**.

# General 2HDM potential

Remarkable enough, one can still **learn many features** of this model **without minimizing the potential explicitly**, for example,

- find **classes symmetries** of the potential and their spontaneous breaking;
- prove existence of **at most two local minima** of the potential;
- explicitly describe the **full phase diagram** and possible **sequences of phase transitions**.

All this can be obtained with the aid of various basis-invariant methods developed in the last years.

# $N$ Higgs doublets: NHDM

The idea of many Higgs “generations” can be pursued further.

The SM Higgs field gives mass to gauge bosons and to all the fermions, resulting in mass hierarchy problem. Maybe a bit of overload for the poor SM Higgs?

One can consider models where different Higgs doublets give masses to different particles with a hope to find a natural explanation to the flavor structure of SM.

Examples: Weinberg’s 3HDM, Adler’s 6HDM, Private Higgs model by Porto and Zee (each fermion couples to its own Higgs doublet), etc.

Very few general results are known. The main direction of research concerns symmetries (in the Higgs and Yukawa sector), their violation and their phenomenological impact.

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# Extra singlets

The Higgs fields don't have to be doublets. They can be of any representation under the  $SU(2) \times U(1)$  gauge group: singlets, triplets, etc. The only requirement is that they should contain a **neutral component**, which can acquire the v.e.v.

The simplest version is a **neutral singlet field**  $\sigma$ , which can be added to the usual Higgs doublet  $\phi$ .

- **Non-minimal supersymmetric models** can contain extra Higgs singlets.
- **“Higgs portal into Hidden valley”**: a class of models in which there exists a whole new sector of matter fields and interactions which are SM-blind, and the communication is achieved via the Higgs sector.

# Extra singlets: Higgs portal into Hidden valley

Suppose that there exist **new matter fields and new gauge interactions**, which are not directly observable with our detectors (“**Hidden valley**”). The full lagrangian is then

$$\mathcal{L} = \mathcal{L}_{our} + \mathcal{L}_{hidden} + \mathcal{L}_{link} .$$

How can we construct  $\mathcal{L}_{link}$ , the link between our world and the hidden valley?

If we insist on the renormalizable theories, there are only two possibilities: **kinetic mixing** between gauge bosons (not discussed here), and the **Higgs portal**.

# Extra singlets: Higgs portal into Hidden valley

Renormalizable theory  $\rightarrow \mathcal{L}_{link}$  cannot have interaction operators of dimension higher than 4.

All terms in the SM lagrangian have dimension 4 except for one:  $-\mu^2(\phi^\dagger\phi)$ . This term also has the only “exterior” mass scale explicitly introduced in SM.

An attractive idea: [what if the SM lagrangian has no scale at all?](#)  
 What we perceive as  $\mu^2$  can be in fact a vacuum expectation value of a new scalar field  $\sigma$ :

$$\eta(\phi^\dagger\phi)\sigma^2 \rightarrow \eta(\phi^\dagger\phi)\langle\sigma^2\rangle \equiv -\mu^2(\phi^\dagger\phi).$$

This field  $\sigma$  must be an EW-singlet, but it can transform non-trivially under the “hidden” interactions  $\rightarrow$  that’s why  $\sigma^2$ .

# Extra singlets: Higgs portal into Hidden valley

In a simplest model, such an interact term after EWSB leads to interactions:

$$\eta(\phi^\dagger\phi)\sigma^2 \rightarrow \eta(2vh + h^2)(2\langle\sigma\rangle h_\sigma + h_\sigma^2).$$

- If  $\langle\sigma\rangle \neq 0$ , then we have **direct mixing**  $h \leftrightarrow h_\sigma$ .

Collider signature:  $gg \rightarrow h \rightarrow h_\sigma \rightarrow$  hidden valley particles.

- If  $\langle\sigma\rangle = 0$ , we still have decays  $h^* \rightarrow h_\sigma h_\sigma$ .

An illustration of the general statement that **the Higgs boson is interesting not on its own but as a window into possible New Physics.**

# Extra triplets

Consider a model with a doublet and a **triplet**:

$$\phi(T = 1/2) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \xi(T = 1) = \begin{pmatrix} \xi^{++} \\ \xi^+ \\ \xi^0 \end{pmatrix}$$

with the potential

$$V = -m^2(\phi^\dagger\phi) - M^2(\xi^\dagger\xi) + \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\xi^\dagger\xi)^2 + \lambda_3(\phi^\dagger\phi)(\xi^\dagger\xi) \\ + \mu \left[ \xi^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^+\phi^0 + \xi^{--}\phi^+\phi^+ \right]$$

with very large  $M^2$ . Both  $\langle\phi^0\rangle = v$  and  $\langle\xi^0\rangle = u$  are non-zero, but

$$u \approx \mu \frac{v^2}{M^2} \text{ is naturally small.}$$

# Extra triplets

The use of the triple is that it can **generate neutrino masses** without right-handed neutrinos:

$$f_{ij} \left[ \xi^0 \nu_i \nu_j + \xi^+ (\nu_i l_j + l_i \nu_j) / \sqrt{2} + \xi^{++} l_i l_j \right] + h.c.$$

Here  $\langle \xi^0 \rangle$  is very small, and so are the neutrino masses. Neutrino masses at the eV scale require  $M \sim 10^{13}$  GeV.

Additional features:

- **small but non-zero lepton number violation**: heavy triple  $\xi^{++}$  can decay into  $\phi^+ \phi^+$  ( $L = 0$ ) and into  $l_i^+ l_j^+$  ( $L = -2$ ).
- A model with two such triplets and with  $CP$ -violation naturally leads to leptogenesis.

# Many other models

A lot of other models of EWSB are being discussed:

- Little Higgs models,
- Technicolor and composite Higgses,
- Higgs-gauge unification models,
- Models with extra dimensions,

and many other.

Hopefully, in a couple of years from now we'll have good clues on where to focus.

## Further reading

- J.D. Wells, “*Lectures on Higgs Boson Physics in the Standard Model and Beyond*”, arXiv:0909.4541.
- E. Accomando et al, “*Workshop on CP Studies and Non-Standard Higgs Physics*”, hep-ph/0608079.
- Ch. Grojean, “*New approaches to electroweak symmetry breaking*”, UFN 177, 3 (2007).