Higgs models: part 1

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The Standard model

The Standard Model (SM) is a gauge theory of electromagnetic, weak and strong interactions of fundamental matter fields (quarks and leptons).

- Gauge principle: the matter fields have internal degrees of freedom, but the physical observables are independent of transformations in this internal space performed independently at each space-time point.
- Forces acting between the matter fields (i.e. gauge interactions) inevitably follow from this requirement.
- The transformation group of SM is $SU(3)_c \times SU(2)_L \times U(1)_Y$ (strong and electroweak interactions).
- Apart from very few issues, the SM describes the vast majority of experimental data extremely well.

Electroweak symmetry

The electroweak symmetry of SM is not manifest but is broken.

- the weak force mediators, $W^{\pm}$ and Z^0 bosons, are different from the EM mediator γ.
- W and Z bosons are massive; their mass put by hand cannot be gauge-invariant.
- chiral fermion masses are also incompatible with the EW invariance.

Spontaneous breaking of the EW symmetry is a vital ingredient of the SM.

We don't know yet how it is realized in nature (minimal Higgs, non-minimal Higgs, Higgsless mechanism etc.) but it must be there.

The abelian Higgs model

QED lagrangian:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi\,,$$

where $D_\mu = \partial_\mu + ieA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Local $U(1)$-symmetry: the lagrangian is invariant under

$$\psi \to e^{-i\alpha(x)}\psi\,, \quad A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)\,.$$

Suppose we want to have QED with massive photon. Adding the photon mass with

$$\mathcal{L}_{mass} = \frac{m_A^2}{2}A_\mu A^\mu$$

does not work because this term is not gauge-invariant!

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Way out: the (abelian) Higgs mechanism .

We introduce a complex scalar field Φ with charge q which feels the gauge interaction and couples to itself:

$$\mathcal{L} = \mathcal{L}_{QED} + (D_\mu \Phi)^*(D^\mu \Phi) - V(\Phi)$$

where $D_\mu = \partial_\mu - iqA_\mu$ and

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4\,, \quad \mu^2,\, \lambda > 0\,.$$

Complex Φ can be parametrize as

$$\Phi = \frac{1}{\sqrt{2}}\phi(x)e^{i\xi(x)},$$

where $\phi(x)$ and $\xi(x)$ are real scalar fields.

The abelian Higgs model

$$\Phi = \frac{1}{\sqrt{2}}\phi(x)e^{i\xi(x)},$$

The scalar potential simplifies to

$$V(\Phi) \to V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4.$$

The ground state (the vacuum) of the model is at

$$\phi(x) = \phi_0 \equiv \sqrt{\frac{\mu^2}{\lambda}}.$$

What about the second field $\xi(x)$?

The abelian Higgs model

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The abelian Higgs model

The other scalar field, $\xi(x)$, enters only via the kinetic term $|D_\mu \Phi|^2$.

$$D_\mu \Phi = \frac{1}{\sqrt{2}} \left[\partial_\mu \phi + i(\partial_\mu \xi - qA_\mu)\phi\right] e^{i\xi(x)} .$$

The field $\xi(x)$ can be gauged away by choosing $\alpha(x) = e\xi(x)/q$ and it disappears from the lagrangian completely. The kinetic term becomes

$$|D_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} q^2 \phi^2 A_\mu A^\mu ,$$

and generates not only kinetic term for the $\phi(x)$, but also $\phi^2(x) A^2(x)$-interaction.

The abelian Higgs model

Expanding the scalar potential at $\phi(x) = \phi_0 + h(x)$:

$$-\mathcal{L}_h = \lambda\phi_0^2 h^2 + \lambda\phi_0 h^3 + \frac{\lambda}{4}h^4 \,,$$

which gives the mass of the Higgs boson $m_h^2 = 2\lambda\phi_0^2 = 2\mu^2$ and cubic (h^3) and quartic (h^4) self-interactions.

We also automatically obtain the "photon" mass $m_A = |q|\phi_0$.

In addition, we have $hA_\mu A^\mu$ and $hhA_\mu A^\mu$ interactions.

The role of gauge symmetry

Looking back at the calculation: what if we didn't have the $U(1)$ gauge symmetry?

There would be no freedom to "gauge away" the field $\xi(x)$.We would still have the global $U(1)$-symmetry with constant phase shift α. We can still spontaneously break the global $U(1)$ symmetry by selecting real vacuum expectation value ϕ_0, but we cannot make $\Phi(x)$ real everywhere.

Small fluctuations would be

$$\Phi(x) = \frac{1}{\sqrt{2}}\phi(x)e^{i\xi(x)} \approx \frac{1}{\sqrt{2}}\left[\phi_0 + h(x) + ig(x)\right] ,$$

where $g(x) = \phi_0 \cdot \xi(x)$.

The role of gauge symmetry

The field $g(x)$ does not disappear from the kinetic term

$$|\partial_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu g)^2 + \dots\,,$$

but it is still absent in the potential $\to$ zero mass. This is the Goldstone boson, the massless boson which appears every time when a global continuous symmetry spontaneuously breaks.

The essence of the Higgs mechanism:
in a gauge theory, the Goldstone boson disappears and reemerges as a longitudinal degree of freedom of the massive gauge boson.

Chiral fermions

In the above toy model we also had fermions described by a Dirac spinor ψ with the explicit mass term $m\bar\psi\psi$.

In terms of two chiral fermions ψ_L and ψ_R:

$$m\bar\psi\psi = m\psi_L^\dagger\psi_R + m\psi_R^\dagger\psi_L\,.$$

The explicit mass term mixes ψ_L and ψ_R.

- It's OK for a vector-field interaction (ψ_L and ψ_R have the same charges).
- It's not OK in gauge theories where the two chiral projections carry different charges: the explicit mass term is not compatible with the gauge symmetry.

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Chiral fermions

The Higgs mechanism solves this problem as well.

No explicit fermion mass term, but there is a Yukawa term

$$\mathcal{L}_f = y_\psi \psi_L^\dagger \Phi \psi_R + c.c.$$

where y_ψ is a dimensionless Yukawa coupling constant and the Higgs field Φ must carry the correct quantum numbers.

After spontaneous symmetry breaking we get

$$\mathcal{L}_f = m_\psi \psi_L^\dagger \psi_R + \frac{m_\psi}{\phi_0} h \psi_L^\dagger \psi_R + c.c.$$

The fermion gets its mass by interacting with the Higgs field: $m_\psi = y_\psi \phi_0$ and get its coupling to the physical Higgs boson proportional to the mass.

The electroweak interactions

- The electroweak $SU(2)_L \times U(1)_Y$ symmetry with massless gauge bosons $A_\mu^{(i)}$, $i = 1, 2, 3$ for $SU(2)$ (with field strength $G_{\mu\nu}$) and B_μ for $U(1)$ (with field strength $F_{\mu\nu}$).
- The chiral fermions interact in different ways. The left fermions ψ_L are $SU(2)$-doublets and therefore interact with $A_\mu^{(i)}$, while the right fermions ψ_R are singlets and do not feel the $SU(2)$-interactions.
- The Higgs field mixes left and right chiral fermions $(\psi_L^\dagger \Phi \psi_R)$, therefore it must be an $SU(2)$-doublet itself:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} .$$

The electroweak lagrangian

The electroweak lagrangian (for a single lepton generation):

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\Phi|^2 - V(\Phi) \\ &+ iL^\dagger D_\mu\gamma^\mu L + ie_R^\dagger D_\mu\gamma^\mu e_R + i\nu_R^\dagger D_\mu\gamma^\mu \nu_R - f_e(L^\dagger\Phi e_R + e_R^\dagger\tilde{\Phi}L)\,.\end{aligned}$$

Gauge covariant derivative

$$D_\mu = \partial_\mu - ig\,T^iA^i_\mu - ig'\frac{Y}{2}B_\mu\,,$$

where g and g' are gauge coupling constants, T^i are $SU(2)$ generators and Y is a hypercharge for a given gauge group representation.

EW symmetry breaking

Spontaneous symmetry breaking essentially as before. The vacuum expectation value of the Higgs doublet is

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v \end{array}\right).$$

- This v.e.v. has definite and non-zero $Y = 1$ and $T^3 = -1/2 \to$ vacuum does not conserve neither electroweak charges nor hyperchange;
- their combination $Q = T^3 + Y/2 = 0 \to$ vacuum conserves the electric charge Q.
- The electroweak symmetry group $SU(2)_L \times U(1)_Y$ is broken down to $U(1)_Q$.

EW symmetry breaking

Small fluctuations around the minimum: among four degrees of freedom

$$\Phi = \langle\Phi\rangle + \frac{1}{\sqrt{2}}\left(\begin{array}{c} \phi_1 + i\phi_2 \\ h + i\phi_3 \end{array}\right).$$

three can be "gauged away" and will reappear as longitudinal components of the three massive gauge bosons. The remaining scalar degree of freedom, h, is the physical Higgs boson.

Masses of the gauge bosons

$$
\begin{aligned}
D_\mu\Phi &= \left(\partial_\mu - \frac{i}{2} g\sigma^i A^i_\mu - \frac{i}{2} g' B_\mu\right)\Phi \\
&\to \frac{i}{2\sqrt{2}}(gA^3_\mu - g'B_\mu)\begin{pmatrix} 0 \\ v \end{pmatrix} - \frac{i}{2\sqrt{2}} g(A^1_\mu - iA^2_\mu)\begin{pmatrix} v \\ 0 \end{pmatrix}. \\
|D_\mu\Phi|^2 &\to \frac{v^2}{8}(gA^3_\mu - g'B_\mu)^2 + \frac{g^2v^2}{8}(A^1_\mu - iA^2_\mu)^2 \\
&= \frac{\bar{g}^2 v^2}{8} Z_\mu Z^\mu + \frac{g^2 v^2}{4} W^-_\mu W^{+\mu}.
\end{aligned}
$$

Here $\bar{g}^2 \equiv g^2 + g'^2$ and the new fields (weak vector bosons) are

$$
Z_\mu = \frac{gA^3_\mu - g'B_\mu}{\bar{g}}, \quad W^\pm_\mu = \frac{A^1_\mu + iA^2_\mu}{\sqrt{2}}
$$

Masses of the gauge bosons

The W and Z bosons are massive:

$$m_Z = \frac{\bar{g}v}{2}\,, \quad m_W = \frac{gv}{2}\,, \quad \frac{m_W}{m_Z} = \frac{g}{\bar{g}} \equiv \cos\theta_W\,.$$

where θ_W is the Weinberg angle.

The combination orthogonal to Z_μ, $A_\mu = (g' A^3_\mu + g B_\mu)/\bar{g}$ is absent. This is the massless photon.

The value of θ_W is the only independent parameter of the EW model. Once it is fixed, the values of g, g', m_W, m_Z, v can be predicted from low energy data (namely the elementary electric charge and the Fermi weak constant G_F). These predictions have been tested experimentally and are found to be in a very good agreement.

Constraints on the Higgs sector

I don't discuss the Higgs boson phenomenology at colliders!

The v.e.v. is known very well, $v = 246$ GeV, but the Higgs field quartic self-coupling λ is unknown $\to$ no prediction for the SM Higgs mass m_h.

Direct searches at the LEP, Tevatron and (soon) the LHC place experimental constraints on the Higgs mass. But apart from them, purely theoretical arguments exist on where the SM Higgs mass can and cannot be.

Two important restrictions are perturbative unitarity and vacuum stability.

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Perturbative unitarity

Consider a high-energy elastic scattering process. Unitarity: scattering amplitude cannot be arbitrarily large, otherwise the outgoing flux will be larger than the incoming flux.

Expanding the scattering amplitude in the partial waves:

$$\mathcal{A} = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) a_\ell \,.$$

where

$$a_\ell = \frac{e^{2i\delta_\ell} - 1}{2i}\,, \quad |\mathrm{Re}\, a_\ell| \le \frac{1}{2}\,.$$

Perturbative unitarity

If the tree-level scattering amplitude for any ℓ gives $|a_\ell| > 1/2$, it means that very strong higher order corrections are to be expected.

The tree-level calculation is unreliable and the perturbative series is poorly convergent. So, the dynamics of scattering is completely modified with respect to the naive perturbative expectation.
We need to check that the tree-level scattering amplitudes do not shoot up in the EW theory.

Perturbative unitarity

Most dangerous is scattering of longitudinally polarized W and Z bosons, e.g. $W_L^+ W_L^- \to W_L^+ W_L^-$. Polarization vectors:

$$e_\pm^\mu = -\frac{1}{\sqrt{2}}(0, \pm, i, 0)\,, \quad e_L^\mu = \frac{1}{M}(p, 0, 0, E)\,.$$

Scattering processes involving e_L tend to rise with energy.

The Higgs mechanism tames this rise. The amplitude stays finite at $s \to \infty$, $\mathcal{A} = -2m_h^2/v^2$, but it can be large. Requiring that $|\mathcal{A}| < 8\pi$ gives

$$m_h < 2\sqrt{\pi} v \approx 870 \text{ GeV}\,.$$

A more accurate coupled-channel analysis reduces this bound to 710 GeV.

Restrictions form high-scale behavior

Loop corrections modify the tree-level scalar potential $\to$ effective λ changes with renormalization scale (RG flow).

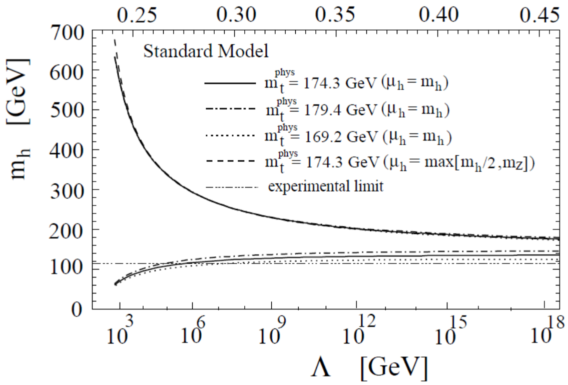
- If λ is large at the EW scale, then it grows further:

$$\frac{d\lambda}{d\log Q^2} \approx \frac{3\lambda^2}{2\pi}$$

  and at some large Q^2 is will violate the perturbativity constraint.
- If λ is too small at the EW scale, the top-quark loop drives λ down; above some scale Q^2, λ becomes negative (vacuum breaks down).

If we insist that the EW theory is valid up to a certain scale Λ^2, we place limits on λ (and therefore on m_h) from above and from below.

Restrictions form high-scale behavior



"Nightmare scenario": if $130 < m_h < 180$ GeV, the EW theory might work up to the Planck scale $\rightarrow$ no clue on where the New Physics is.

EWSB without EWSB

Elitzur theorem (1975): it is impossible to spontaneously break a local symmetry.

In fact, we break the local symmetry by choosing a gauge fixing procedure and break spontaneously only the global symmetry.

It implies that EW symmetry breaking is not really a necessary physical phenomenon in transition from the "EW-symmetric" to the Higgs phase.

One should be able to reformulate the EW theory without explicitly referring to the EW symmetry breaking.

EWSB without EWSB

Consider again the Abelian Higgs model with small fluctuations near the vacuum:

$$\Phi(x) = \frac{1}{\sqrt{2}}\phi(x)e^{ig(x)/\phi_0} \approx \frac{1}{\sqrt{2}}\left[\phi_0 + h(x) + ig(x)\right] ,$$

and write down the terms up to quadratic:

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 + \frac{q^2\phi_0^2}{2}\left(A_\mu - \frac{1}{q\phi_0}\partial_\mu g\right)^2 ,$$

EWSB without EWSB

Let's introduce the gauge invariant field

$$B_\mu \equiv A_\mu - \frac{1}{q\phi_0}\partial_\mu g\,, \quad F^{(A)}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = F^{(B)}_{\mu\nu}\,.$$

Now the quadratic lagrangian

$$\mathcal{L}_2 = -\frac{1}{4}F^2_{\mu\nu} + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 + \frac{q^2\phi_0^2}{2}B_\mu B^\mu$$

contains degrees of freedom which are explicitly $U(1)$-gauge invariant.

Now, there is no freedom of gauge transformation left. The issue of (spontaneous) breaking of gauge symmetry becomes redundant, because there is no gauge symmetry anymore.

Further reading

A typical flow of topics:

- real scalar field: breaking of a discrete symmetry $\phi(x) \to -\phi(x)$.
- compex field: breaking of a global $U(1)$-symmetry $\to$ Goldstone boson.
- real interacting fields: breaking of a non-abelian global symmetry.
- general case of spontaneous of global symmetries: Goldstone theorem.
- abelian Higgs model: breaking of the $U(1)$ gauge symmetry.
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Further reading

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