BEYOND THE STANDARD MODEL

PHENOMENOLOGY OF SUPERSYMMETRY

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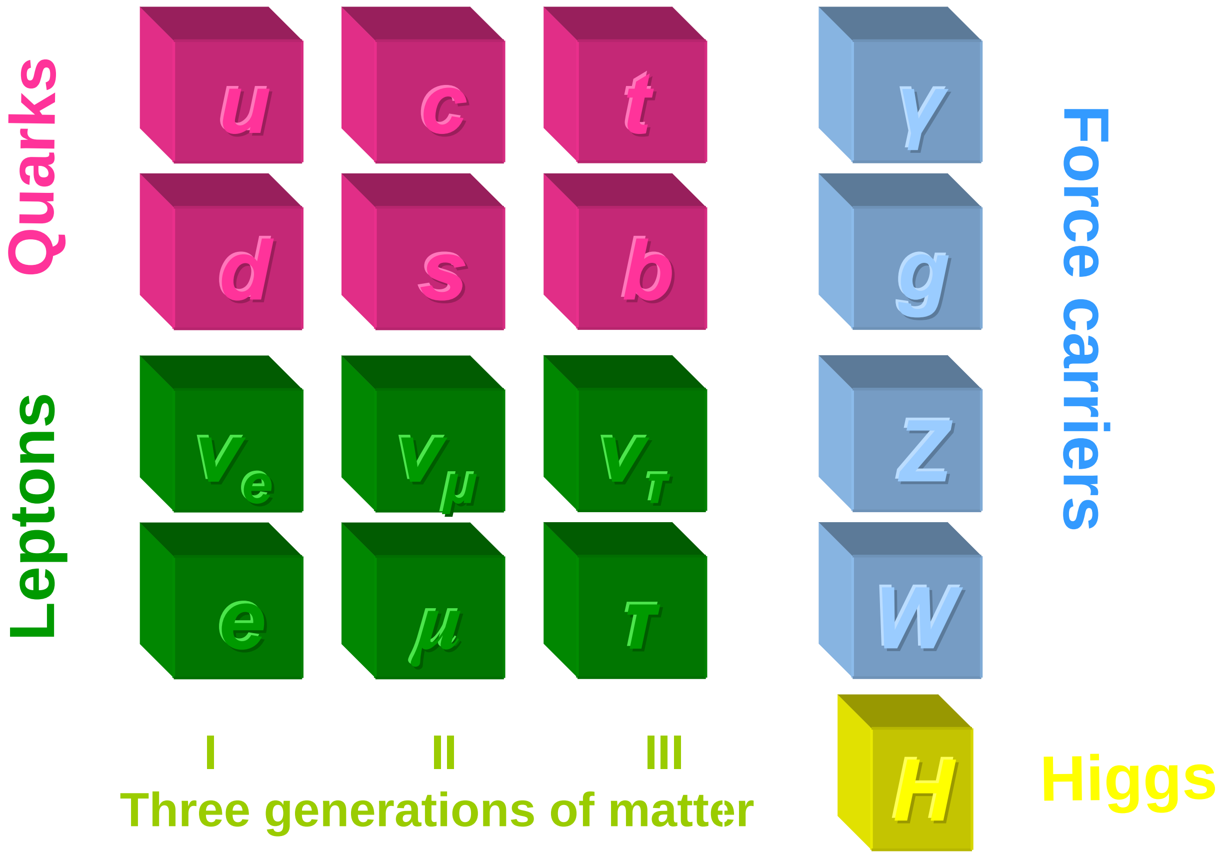
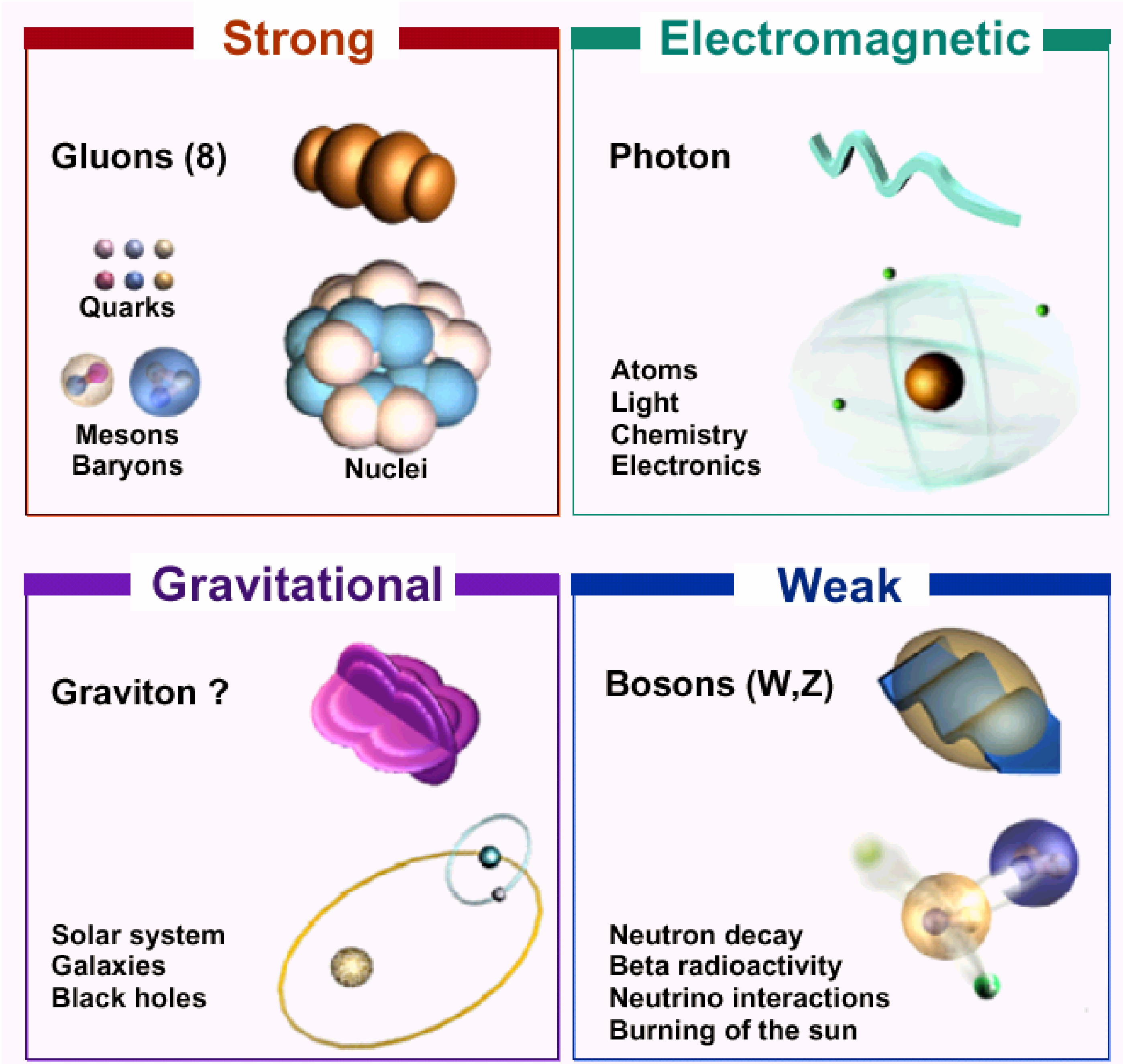
Fundamental Particles





Strong
Gluons (8)
Quarks
Mesons
Baryons
Nuclei
Electromagnetic
Photon
Atoms
Light
Chemistry
Electronics
Gravitational
Graviton ?
Solar system
Galaxies
Black holes
Weak
Bosons (W,Z)
Neutron decay
Beta radioactivity
Neutrino interactions
Burning of the sun

The Standard Model

- Standard Model Lagrangian

$$L = L_{gauge} + L_{Yukawa} + L_{Higgs}$$

- Gauge interactions (kinetic terms for the gauge fields, quarks, leptons and Higgs bosons; self-interactions of the gauge fields; interactions of the gauge fields and Higgs bosons)

$$\begin{aligned} L_{gauge} &= -\tfrac{1}{4} G^{a}_{\mu\nu} G^{a}_{\mu\nu} - \tfrac{1}{4} W^{i}_{\mu\nu} W^{i}_{\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B_{\mu\nu} \\ &+ i\overline{L}_{\alpha}\gamma^{\mu} D_{\mu} L_{\alpha} + i\overline{Q}_{\alpha}\gamma^{\mu} D_{\mu} Q_{\alpha} + i\overline{E}_{\alpha}\gamma^{\mu} D_{\mu} E_{\alpha} \\ &+ i\overline{U}_{\alpha}\gamma^{\mu} D_{\mu} U_{\alpha} + i\overline{D}_{\alpha}\gamma^{\mu} D_{\mu} D_{\alpha} + (D_{\mu} H)^{\dagger}(D_{\mu} H) \end{aligned}$$

The Standard Model

- Yukawa interactions (interactions of quark and leptons with the Higgs boson)

$$L_{Yukawa} = y^{L}_{\alpha\beta}\overline{L}_{\alpha}E_{\beta}H + y^{D}_{\alpha\beta}\overline{Q}_{\alpha}D_{\beta}H + y^{U}_{\alpha\beta}\overline{Q}_{\alpha}U_{\beta}\widetilde{H}$$

$$\widetilde{H} = i\tau_2 H^{\dagger}$$

- Scalar potential (mass term and self-interaction of the Higgs boson)

$$L_{Higgs} = -V = m^2 H^{\dagger}H - \frac{\lambda}{2}(H^{\dagger}H)^2$$

The Standard Model: drawbacks

- Large number of free parameters:
  - gauge coupling constants g_s , g , g'
  - 3×3 matrices of Yukawa coupling constants
  - coupling constant of the Higgs self-interaction
  - the Higgs mass parameter
  - mixing angles and phases

  How one can reduce the number of parameters ?

- The choice of the gauge group:

  why there are three independent symmetry groups ?

$$SU(3)_C \times SU(2)_{EW} \times U(1)_Y$$

The Standard Model: drawbacks

- The unification of the strong and electroweak interactions is formal
- Why the «strong» interactions are strong and «weak» ones are weak ?
- Why there are 3 generations of the matter fields ?
- The origin of particle masses: why are particles massive ?
- Why the top-quark is heavy and leptons are light ?
- Is the Higgs boson a fundamental particle ?
  What is the mass of the Higgs boson ?
- Why the proton charge is equal to the elctron charge ?
- How can we include gravity into the theory ?
- The Standard Model has no answers

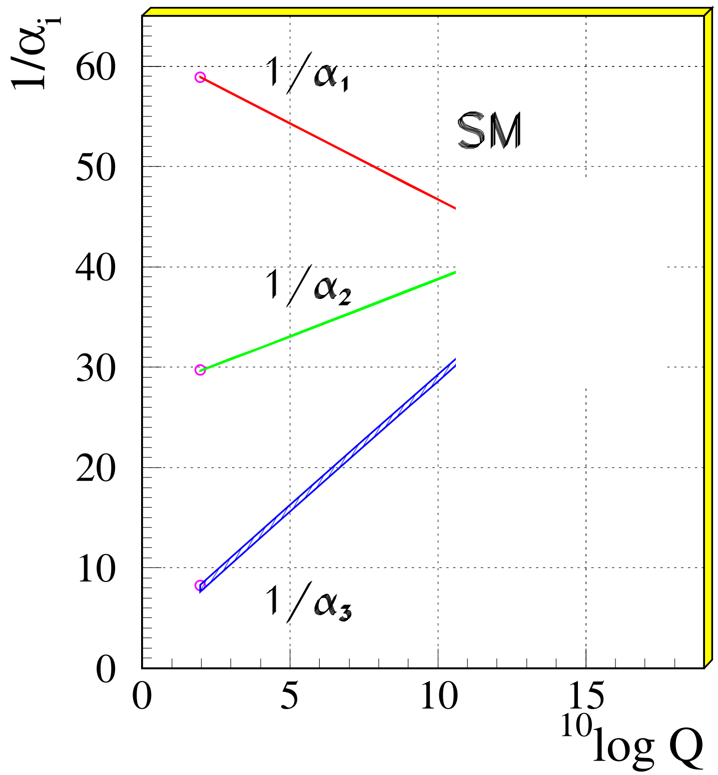
The Standard Model: what to do?

- CONCLUSION: The Standard Model is an effective theory valid within a certain approximation
- WHAT TO DO: consider *more symmetric* theories

- Examples:
  - Grand Unification Theories: The strong, weak and electromagnetic interactions are described by one symmetry group
  - Supersymmetry: Bosons and fermions are described in a common way.

Grand Unification

- The idea of unification is based on the observation that three gauge couplings tends to the same point at high energy



- Evolution equations (SM)

$$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi} = \frac{g_i^2}{16\pi^2}, \quad t = \log \frac{Q^2}{\mu^2}$$

$$\frac{1}{\tilde{\alpha}_i} = \frac{1}{\tilde{\alpha}_{0i}} - b_i t$$

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}$$

Hierarchy problem

- Hierarchy problem

  Why there are very different energy scales ?

  - Electroweak symmetry breaking scale ($M_W \sim 100\ GeV$)
  - Grand Unification scale ($M_{GUT} \sim 10^{15\text{-}16}\ GeV$)

    or Plank scale ($M_{Pl} \sim 10^{19}\ GeV$)

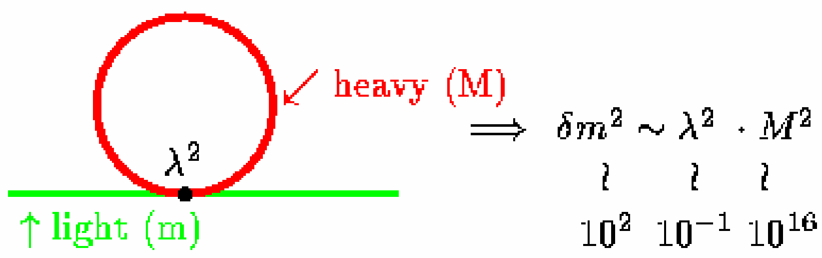
- Possible solution: to postulate the hierarchy.

  Very unnatural !

Hierarchy problem

- Another side of the problem: the hierarchy is destroyed by the radiative corrections

Consider the correction
to the light Higgs boson mass



$$m_H \sim v \sim 10^2\ GeV$$

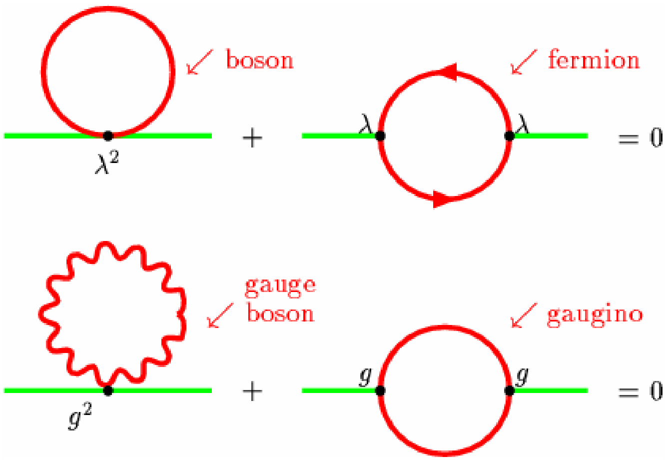
$$M_\Sigma \sim V \sim 10^{16}\ GeV$$

Even if the hierarchy was postulated it is destroyed by radiative corrections (unless they cancel up to 10^{-14})

Hierarchy problem

- Supersymmetry can help to solve the hierarchy problem

- Let us add a «superpartner» -
  a particle with the same mass but with a different spin.
  Then the divergency cancells.



- The «accuracy» of cancellation is controlled by the mass-squared difference.

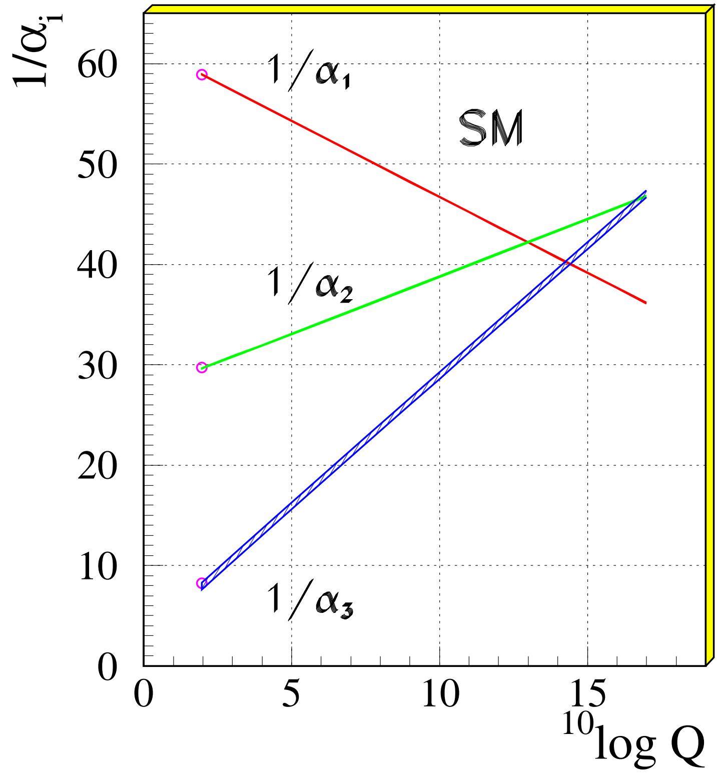
$$m^2_{boson} - m^2_{fermion} = M^2_{SUSY}$$

- If the correction is not larger than the mass itself then we have

$$\delta m_h^2 \sim g^2 M^2_{SUSY} \sim m_h^2 \sim 10^4 GeV \quad \Rightarrow \quad M_{SUSY} \sim 10^3 GeV$$

Grand Unification

- However, there is no Grand Unification at high energies if we use the Standard Model evolution equations for the gauge couplings



- Evolution equations (MSSM)

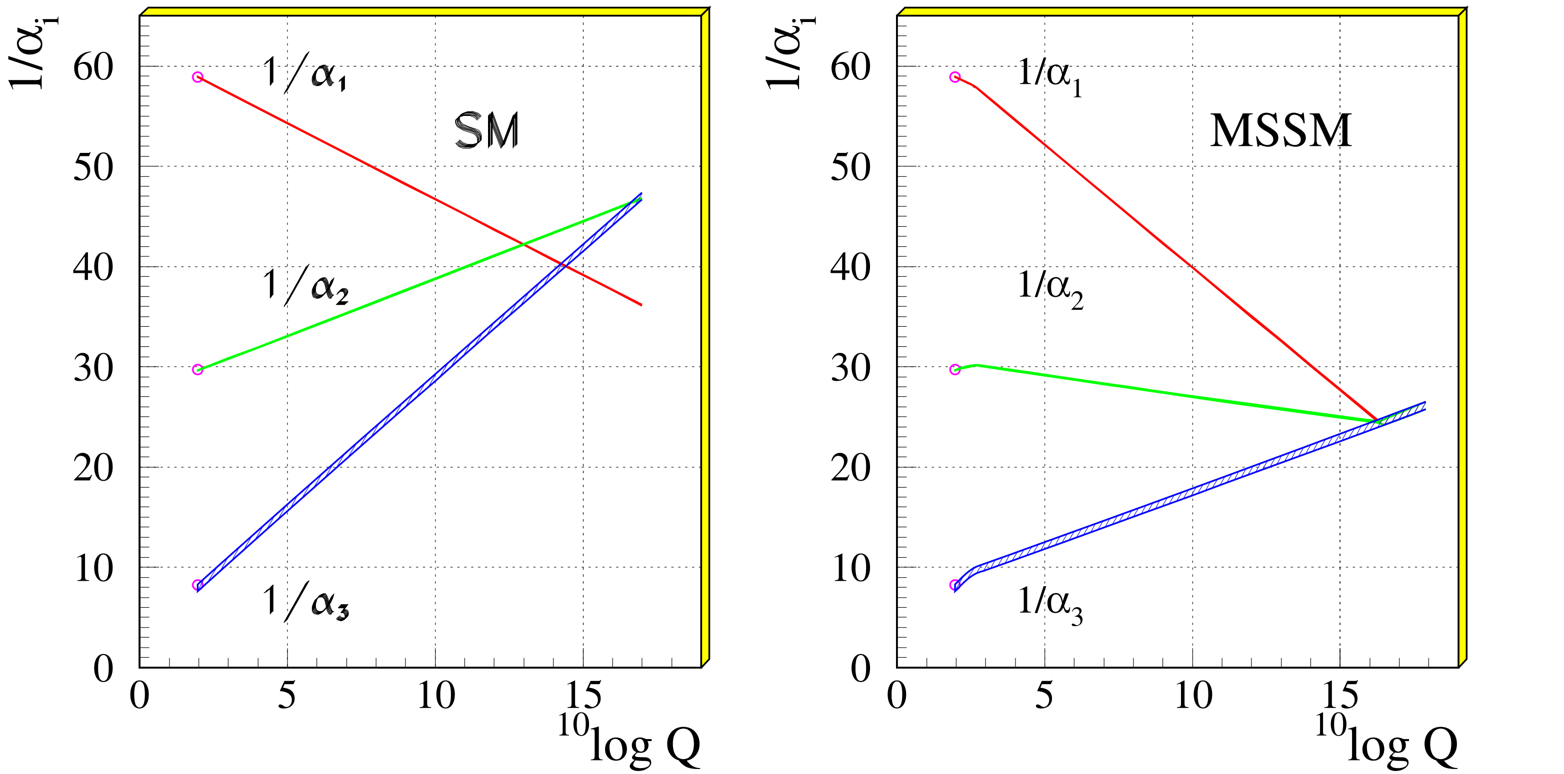
$$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi} = \frac{g_i^2}{16\pi^2}, \quad t = \log\frac{Q^2}{\mu^2}$$

$$\frac{1}{\tilde{\alpha}_i} = \frac{1}{\tilde{\alpha}_{0i}} - b_i t$$

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix}$$

Grand Unification

- In the Minimal supersymmetric Standard Model the gauge coupling constants do unify !



Grand Unification

- CONCLUSION: we need supersymmetry for unification

- Initial conditions at low energy are known ('93)

$$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$$

$$\sin^2\theta_{\overline{MS}} = 0.23146 \pm 0.00017$$

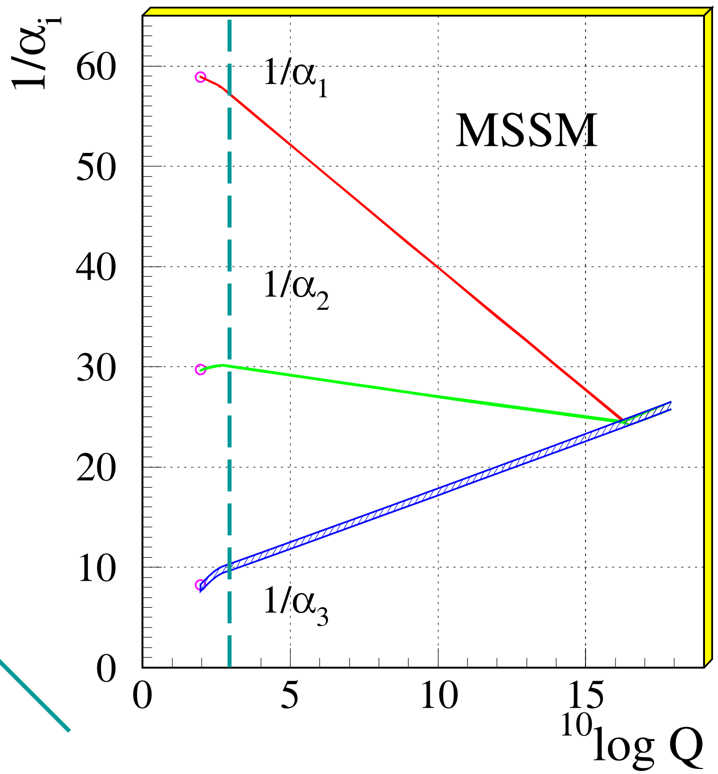
$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

then we calculate

$$M_{SUSY} = 10^{3.4 \pm 0.9 \pm 0.4}\ \mathrm{GeV}$$

$$M_{GUT} = 10^{15.8 \pm 0.3 \pm 0.1}\ \mathrm{GeV}$$

$$\alpha^{-1}_{\mathrm{GUT}} = 26.3 \pm 1.9 \pm 1.0$$



- The scale of supersymmetry breaking is $\sim 1\ TeV$

Supersymmetry: motivations

- Consistency of Grand Unification theory :
  unification of gauge coupling constants
- Solution to the hiearchy problem
- Supersymmetry populates «The Great Desert» : it predicts new particles and their spectrum
- Supersymmetry suggest a solution of the Dark Matter problem
- Radiative electroweak symmetry breaking.
  The Higgs boson mass is calculable.
- Supersymmetry can be tested experimentally
- Unification of particle physics and gravity (supergravity).
- SUSY is the most popular idea beyond the Standard Model

Supersymmetry

- Supersymmetry is a symmetry between bosons and fermions

$$Q\,|\,boson\rangle = |\,fermion\rangle$$
$$Q\,|\,fermion\rangle = |\,boson\rangle$$

- Q commutes with energy and momentum operators:

$$[Q, P] = [Q, E] = 0$$

- The crucial consequence is that particles related by supersymmetry transformations have the same mass

- In particle physics N=1 supersymmetry is used.

Superspace and superfields

- Superspace – is an extension of Minkowski space by addition of a pair of grassmanian (anticommuting) coordinates

$$x_\mu \to x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}$$

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0, \quad \theta_\alpha^2 = 0, \quad \theta_\beta^2 = 0, \quad \alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$$

- Superfields are functions defined on superspace. Their Taylor expansion is finite. The coefficients are usual fields, called the component fields

$$\begin{aligned} F(x,\theta,\bar{\theta}) = f(x) + i\theta\chi(x) + \overline{\theta\chi}(x) \\ +\theta\theta m(x) + \bar{\theta}\bar{\theta} n(x) + \theta\sigma^\mu\bar{\theta} v_\mu(x) \\ +\theta\theta\overline{\theta\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta} d(x) \end{aligned}$$

Superspace and superfields

- The simplest example is a chiral superfield, defined as

$$\bar{D}_{\dot{\alpha}} F(x,\theta,\bar{\theta}) = 0 \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i(\theta\sigma^{\mu})_{\dot{\alpha}} \partial_{\mu}$$

- The expansion in Taylor series has the form

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \qquad y = x + i\theta\sigma\bar{\theta}$$

$$= A(x) + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x)$$

$$+\sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta} + \theta\theta F(x)$$

- *A(x)* – complex scalar field (2 bosonic d.o.f.),
  ψ(x) – Weyl spinor field (2 fermionic d.o.f.)

- *F(x),* the auxiliary field is unphysical and can be eliminated

Superspace and superfields

- The anti-chiral superfield is defined as

$$D_\alpha \Phi^\dagger = 0 \qquad D_\alpha = \frac{\partial}{\partial \theta_\alpha} + i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu$$

- The chiral and antichiral superfields are used to describe matter

- To describe gauge interactions we need a real vector superfield

$$\begin{aligned} V(x,\theta,\bar{\theta}) = C(x) &+ i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) \\ &+ i\theta\theta M(x) - i\bar{\theta}\bar{\theta}M^+(x) - \theta\sigma^\mu\bar{\theta}V_\mu(x) \\ &+ i\theta\theta\bar{\theta}[\bar{\lambda}(x) + i\bar{\sigma}^\mu\partial_\mu\chi(x)] - i\bar{\theta}\bar{\theta}\theta[\lambda(x) + i\sigma^\mu\partial_\mu\bar{\chi}(x)] \\ &+ \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \tfrac{1}{2}\Box C(x)] \end{aligned}$$

Superspace and superfields

- In a particular gauge (Wess-Zumino gauge) one has

$$V = -\theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x)$$

$$V^{2} = -\tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\, v_{\mu}(x)v^{\mu}(x)$$

$$V^{3} = 0$$

- The field strength tensor

$$W_{\alpha} = -\frac{1}{4}\bar{D}^{2}D_{\alpha}V \qquad W_{\dot{\alpha}} = -\frac{1}{4}D^{2}D_{\dot{\alpha}}V$$

$$W_{\alpha} = -i\lambda_{\alpha} + \theta_{\alpha}D - \tfrac{i}{2}(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)_{\alpha}F_{\mu\nu} + \theta^{2}\sigma^{\mu}\partial_{\mu}\bar{\lambda}$$

$$F_{\mu\nu} = \partial_{\mu}v_{\nu} - \partial_{\nu}v_{\mu}$$

Supersymmetric lagrangians

- The action is the integral over superspace

$$Action = \int d^4x\, \mathcal{L} \quad \Longrightarrow \quad \int d^4x\, d^4\theta\, \mathcal{L}$$

- SUSY invariant lagrangian

$$L = \int d^2\theta d^2\overline{\theta}\, \Phi_i^+ \Phi_i + \int d^2\theta\, (\lambda_i \Phi_i + \tfrac{1}{2} m_{ij} \Phi_i \Phi_j + \tfrac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k) + h.c.]$$

- In components one has

$$L = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i - \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j$$
$$- y_{ijk} \psi_i \psi_j A_k - y_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - V(A_i, A_j)$$

Supersymmetric lagrangians

- Gauge and SUSY invariant lagrangian

$$\mathcal{L}_{SUSY\ YM} = \tfrac{1}{4}\int d^2\theta\ \mathrm{Tr}(W^\alpha W_\alpha) + \tfrac{1}{4}\int d^2\bar\theta\ \mathrm{Tr}(\overline{W}^\alpha \overline{W}_\alpha)$$

$$+\int d^2\theta d^2\bar\theta\ \overline{\Phi}_{ia}(e^{gV})^a_b \Phi^b_i + \int d^2\theta\ \mathcal{W}(\Phi_i) + \int d^2\bar\theta\ \overline{\mathcal{W}}(\overline{\Phi}_i)$$

- In components one has

$$L_{SUSY\ YM} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - i\lambda^a \sigma^\mu D_\mu \overline{\lambda}^a + \tfrac{1}{2} D^a D^a$$

$$+(\partial_\mu A_i - igv^a_\mu T^a A_i)^\dagger (\partial_\mu A_i - igv^a_\mu T^a A_i) - i\overline{\psi}_i \overline{\sigma}^\mu (\partial_\mu \psi_i - igv^a_\mu T^a \psi_i)$$

$$-D^a g A_i^\dagger T^a A_i - i\sqrt{2} g A_i^\dagger T^a \lambda^a \psi_i + i\sqrt{2} g \overline{\psi}_i T^a \overline{\lambda}^a A_i + F_i^\dagger F_i$$

$$+\frac{\partial \mathcal{W}}{\partial A_i} F_i + \frac{\partial \overline{\mathcal{W}}}{\partial A_i^\dagger} F_i^\dagger - \tfrac{1}{2}\frac{\partial^2 \mathcal{W}}{\partial A_i \partial A_j}\psi_i \psi_j - \tfrac{1}{2}\frac{\partial^2 \overline{\mathcal{W}}}{\partial A_i^\dagger \partial A_j^\dagger}\overline{\psi}_i \overline{\psi}_j$$

Supersymmetric lagrangians

- The scalar potential
  - F-term (from SUSY invariant part of the lagrangian)
  - D-term (from gauge invariant part of the lagrangian)

$$\mathrm{V} = \frac{1}{2} D^a D^a + F_i^\dagger F \qquad\qquad D^a = -g A_i^\dagger T^a A_i, \quad F_i = -\frac{\partial \mathcal{W}}{\partial A_i}$$

- The scalar potential is not arbitrary, it is fixed by supersymmetry

- The lagrangian is constructed using only symmetry considerations. One has to choose matter fields and gauge fields

Minimal SUSY SM (MSSM)

- In supersymmetric theories the number of bosonic degrees of freedom is equal to the number of fermionic degrees of freedom

- In the Standard Model we have
  - 28 bosonic degrees of freedom :

$$(4 + 8) \times 2 \quad + \quad 2 \times 2$$

vector fields (γ,Z,W^+,W^-, gluons) — Higgs boson

  - 90 (96) fermionic degrees of freedom:

$$(6 \times 3 + 3) \times 4 \; + \; 3 \times 2 \; (4)$$

quarks and charged leptons — neutrinos

- The Standard Model is not supersymmetric

	Bosons	Fermions	SU(3)	SU(2)	U(1)
Matter fields					
L_i		leptons $L_i = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	-1
E_i		$E_i = e_R$	1	1	2
Q_i		quarks $Q_i = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	1/3
U_i		$U_i = u_R$	3*	1	-4/3
D_i		$D_i = d_R$	3*	1	2/3
Gauge fields					
G^a	gluons g^a		8	0	0
V^k	$W^{\pm}, Z$-bosons		1	3	0
V'	photon γ		1	1	0
Higgs field					
H	Higgs boson $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$		1	2	-1

Minimal SUSY SM (MSSM)

- In order to supersymmetrize the Standard Model one has to add new particles (superpartners)
  - In the Standard Models there are no fermions with quantum numbers of gauge bosons
  - The Higgs and lepton doublets have the same quantum numbers (1,2,-1). Can they be superpartners?
- One has to add the second Higgs doublet
- Fermion masses (up and down quarks).

  Yukawa interactions in the SM → superpotential in the MSSM

$$\mathcal{L}_{Yukawa} = y^L_{\alpha\beta}\overline{L}_\alpha E_\beta H + y^D_{\alpha\beta}\overline{Q}_\alpha D_\beta H + y^U_{\alpha\beta}\overline{Q}_\alpha U_\beta \widetilde{H}$$

$$\tilde{H} = i\tau_2 H^\dagger$$

	Bosons	Fermions	SU(3)	SU(2)	U(1)
Matter fields					
L_i	sleptons $\tilde{L}_i = \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L$	leptons $L_i = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	-1
E_i	$\tilde{E}_i = \tilde{e}_R$	$E_i = e_R$	1	1	2
Q_i	squarks $\tilde{Q}_i = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L$	quarks $Q_i = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	1/3
U_i	$\tilde{U}_i = \tilde{u}_R$	$U_i = u_R$	3*	1	-4/3
D_i	$\tilde{D}_i = \tilde{d}_R$	$D_i = d_R$	3*	1	2/3
Gauge fields					
G^a	gluons g^a	gluino $\tilde{g}^a$	8	0	0
V^k	$W^{\pm}, Z$-bosons	wino $\tilde{W}^{\pm}$, zino $\tilde{Z}$,	1	3	0
V'	photon γ	photino $\tilde{\gamma}$	1	1	0
Higgs fields					
H_1	Higgs boson $H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix}$	higgsino $\tilde{H}_1 = \begin{pmatrix} \tilde{H}_1^+ \\ \tilde{H}_1^0 \end{pmatrix}$	1	2	-1
H_2	Higgs boson $H_2 = \begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix}$	higgsino $\tilde{H}_2 = \begin{pmatrix} \tilde{H}_2^0 \\ \tilde{H}_2^- \end{pmatrix}$	1	2	1

MSSM Lagrangian

- MSSM lagrangian

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa} + \mathcal{L}_{SoftBreaking}$$

- Yukawa interactions (superpotential)

$$\mathcal{W}_R = y_U Q_L H_2 U_R + y_D Q_L H_1 D_R + y_L L_L H_1 E_R + \mu H_1 H_2$$

- In components this will lead to the Standard Model Yukawa interactions + interactions with superpartners

MSSM Lagrangian

- Supersymmetry allow also the following terms in the superpotential

$$\mathcal{W}_{NR} = \lambda_L L_L L_L E_R + \lambda_L^{'} L_L Q_L D_R + \mu^{'} L_L H_2 + \lambda_B U_R D_R D_R$$

- They break baryon and lepton numbers and are absent in the Standard Model

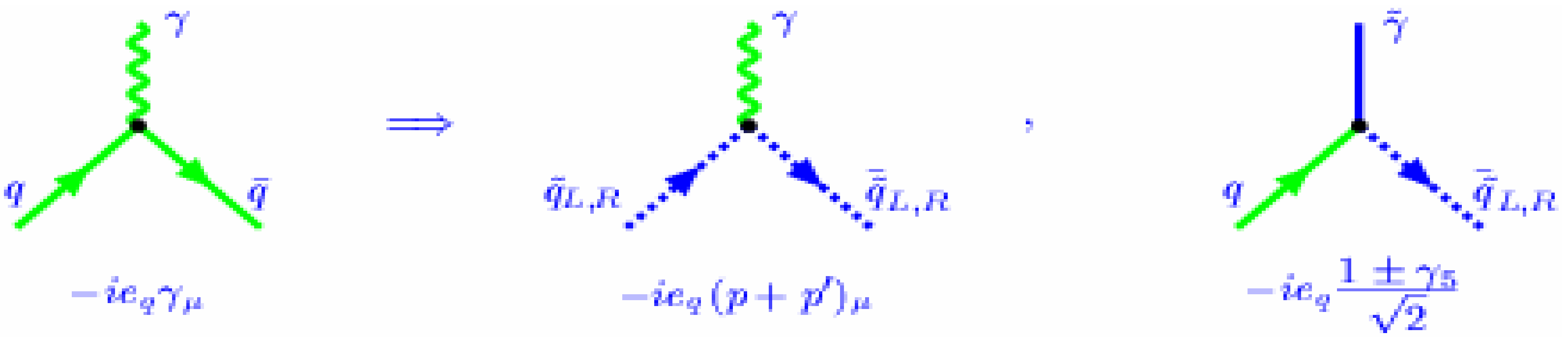
- To get rid of them one has to introduce a new symmetry – R-parity

- All the Standard Model particles have R= +1, and superpartners have R= -1.

$$R = (-1)^{3(B-L)+2S}$$

MSSM Lagrangian

- Consequences of R-parity conservation:
  - Interactions of particles and superpartners are the same (just replace two of the particles in the interaction vertex by superpartners)



  - Superpartners are created in pairs
  - The lightest supersymmetric particle is stable !

Breaking of supersymmetry

- Since superpartners are not observed, in nature supersymmetry can be realised as broken symmetry
- In the MSSM the soft supersymmetry breaking mechanism is used. This can be parametrized by additional terms in the lagrangian
  - The mass terms for the scalar components of chiral superfiels $m_{ij} A_i^* A_j$
  - The mass terms for the fermion components of vector superfiels $M\lambda\lambda$
  - Bilenear softsupersymetry breaking term $B_{ij} \mu_{ij} A_i A_j$
  - Trilinear soft supersymetry breaking terms $A_{ijk} \lambda_{ijk} A_i A_j A_k$

- Supersymmetry is broken since components of the same superfield have different masses

Breaking of supersymmetry

- Part of the MSSM lagrangian responsible for supersymmetry breaking

$$-L_{SoftBreaking} = \sum_{scalars} m_i^2 |A_i|^2 + \sum_{gauge} M_i(\lambda_i\lambda_i + \bar{\lambda}_i\bar{\lambda}_i)$$
$$+A_U y_U Q_L H_2 U_R + A_U y_D Q_L H_1 D_R + A_U y_L L_L H_1 E_R + B\mu H_1 H_2$$

- Too many free parameters (more than a hundred !)

- Now one can calculate the mass spectrum of superparticles

- Later we will see how to reduce the number of parameters

Higgs bosons in the MSSM

- At the tree level the MSSM Higgs potential has the form

$$V_{tree}(H_1,H_2)=m_1^2\,|H_1|^2+m_2^2\,|H_2|^2-m_3^2(H_1H_2+h.c.)$$
$$+\frac{g^2+g'^2}{8}(|H_1|^2-|H_2|^2)^2+\frac{g^2}{2}|H_1^+H_2|^2$$

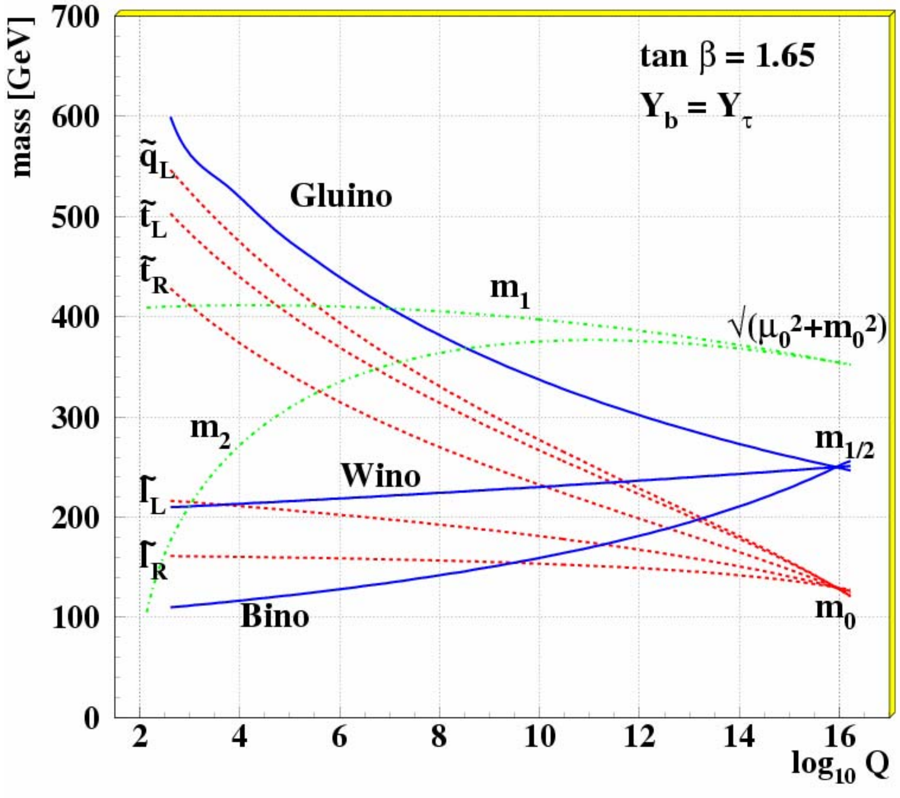
Note: the Higgs self-interaction coupling constant is fixed and is determined by the gauge interactions, this case differs from the Standard Model.

- The MSSM Higgs potential is positively defined and has no non-trivial non-zero minimum.

Higgs bosons in the MSSM

- Running of the Higgs masses leads to the phenomena known as radiative electroweak symmetry breaking.

$$V_{tree}(H_1, H_2)$$
$$= m_1^2 |H_1|^2 - |m_2^2| |H_2|^2$$
$$-m_3^2 (H_1 H_2 + h.c.)$$
$$+ \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2$$



- Возникают условия для спонтанное нарушение электрослабой симметрии

Higgs bosons in the MSSM

- The physical spectrum of the MSSM Higgs sector consists of 5 states:

$$G^0 = -\cos\beta P_1 + \sin\beta P_2 \qquad \textit{Goldstone boson} \rightarrow Z_0$$

$$A = \sin\beta P_1 + \cos\beta P_2 \qquad \textit{Neutral}\ CP = -1\ \textit{Higgs}$$

$$G^+ = -\cos\beta (H_1^-)^* + \sin\beta H_2^+ \qquad \textit{Goldstone boson} \rightarrow W^+$$

$$H^+ = \sin\beta (H_1^-)^* + \cos\beta H_2^+ \qquad \textit{Charged Higges}$$

$$h = -\sin\alpha S_1 + \cos\alpha S_2 \qquad \text{SM}\ \textit{Higgs boson}\ CP = 1$$

$$H = \cos\alpha S_1 + \sin\alpha S_2 \qquad \textit{Extra heavy Higgs boson}$$

- Compare to the Standard Model with 1 Higgs boson.

Higgs bosons in the MSSM

- One can calculate the Higgs masses diagonalizing corresponding mass matrices.

- Masses of the CP-odd and charged Higgs bosons

$$m_A^2 = m_1^2 + m_2^2$$
$$m_{H^\pm}^2 = m_A^2 + M_W^2$$

- Masses of the CP-even Higgs bosons

$$m_{h,H}^2 = \frac{1}{2}[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}]$$

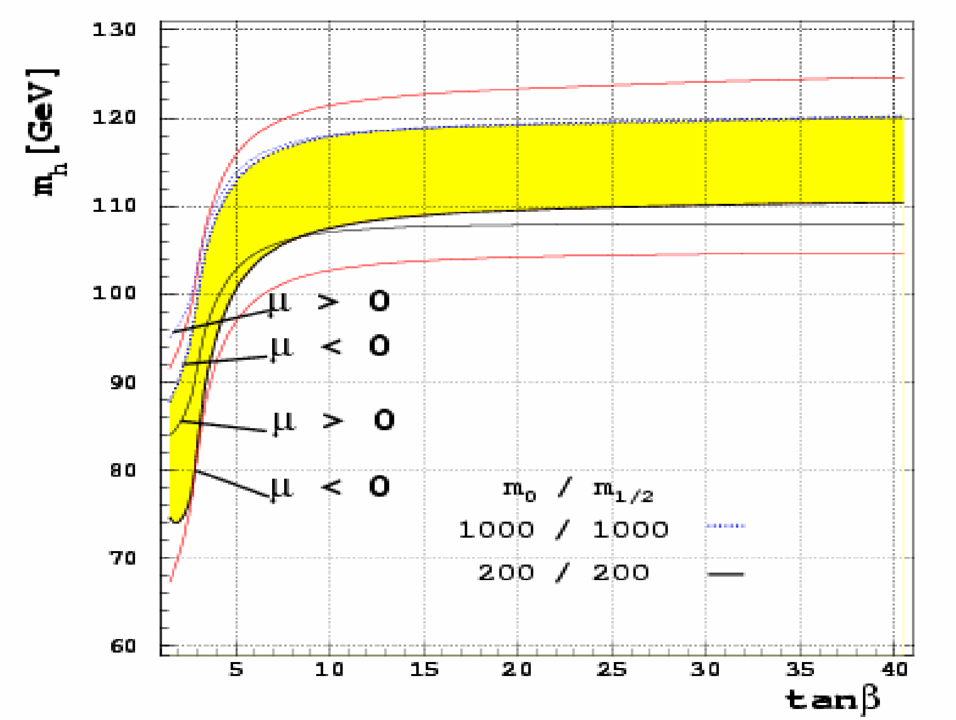
- If $m_A \gg M_Z$, the lightest Higgs boson is lighter than Z-boson !

$$m_h \approx M_Z |\cos 2\beta| < M_Z$$

Higgs bosons in the MSSM

- The inequality $m_h \approx M_Z |\cos 2\beta| < M_Z$ is spoiled by radiative corrections

$$m_h^2 \approx M_Z^2 \cos^2 2\beta$$
$$+ \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4}$$
$$+2 \; loops$$



- 1-loop contribution is very large and positive
- 2-loop contribution is much smaller and negative

Constrained MSSM

- Parameters of the Minimal Supersymmetric Standard Model
  - Gauge cuopling constants α_i, i=1,2,3
  - Yukawa coupling constants $y_{ab}^{k},\ k = U, D, L, (E)$
  - Higgs mixing parameter μ
  - Soft supersymmetry breaking parameters

- The Higgs self-interaction coupling is not arbitrary, it is fixed by supersymmetry.

$$\lambda = \frac{g^2 + g'^2}{8}$$

- The main uncertainty is due to the soft supersymmetry breaking parameters

Constrained MSSM

- Universality hypothesis: soft supersymmetry breaking parameters unify at the scale of Grand Unification

$$-L_{SoftBreaking} = m_0^2 \sum_{scalars} |A_i|^2 + m_{1/2} \sum_{gauge} (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i)$$

$$+A\left(y_t Q_L H_2 U_R + y_b Q_L H_1 D_R + y_L L_L H_1 E_R\right) + B\mu H_1 H_2$$

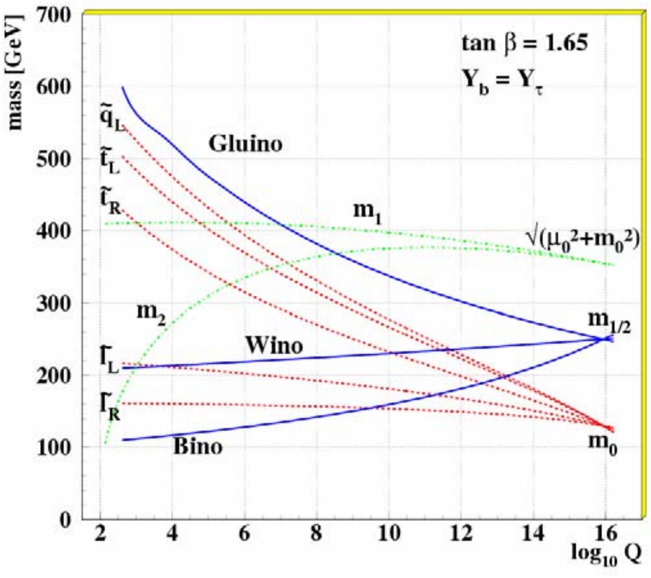
- As a result, MSSM has

  5 free parameters

  $\mu,\ A,\ m_0,\ m_{1/2},\ B(\tan\beta)$

  while the Standard Model has 2 ones

  $m,\ \lambda$



Constrained MSSM

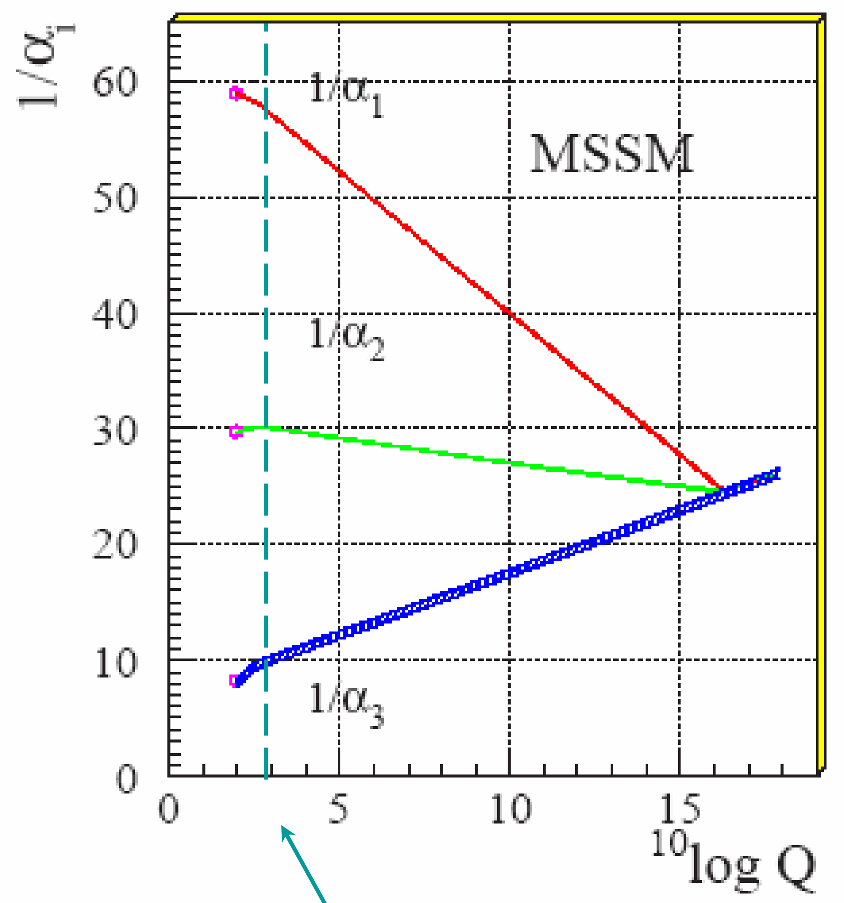
- To make prediction one can choose a certain way
  - Take low-energy values of parameters as input (superpartners masses, mixing parameters, etc.) and then calculate observables as functions of these values.
  - Take high-energy values of parameters as input, then using evolution equations find their low-energy values, calculate the mass spectrum, and then calculate observables. All the calculation now uses a small number of free parameters.

“Experimental” data are sufficient
to find allowed values of initial parameters

Constrained MSSM

- Choice of constraints
  - Unification of the gauge coupling constants

    It is one of the crucial constraints and fixes the scale of supersymmetry breaking.
  - Masses of superpartners are in the TeV region

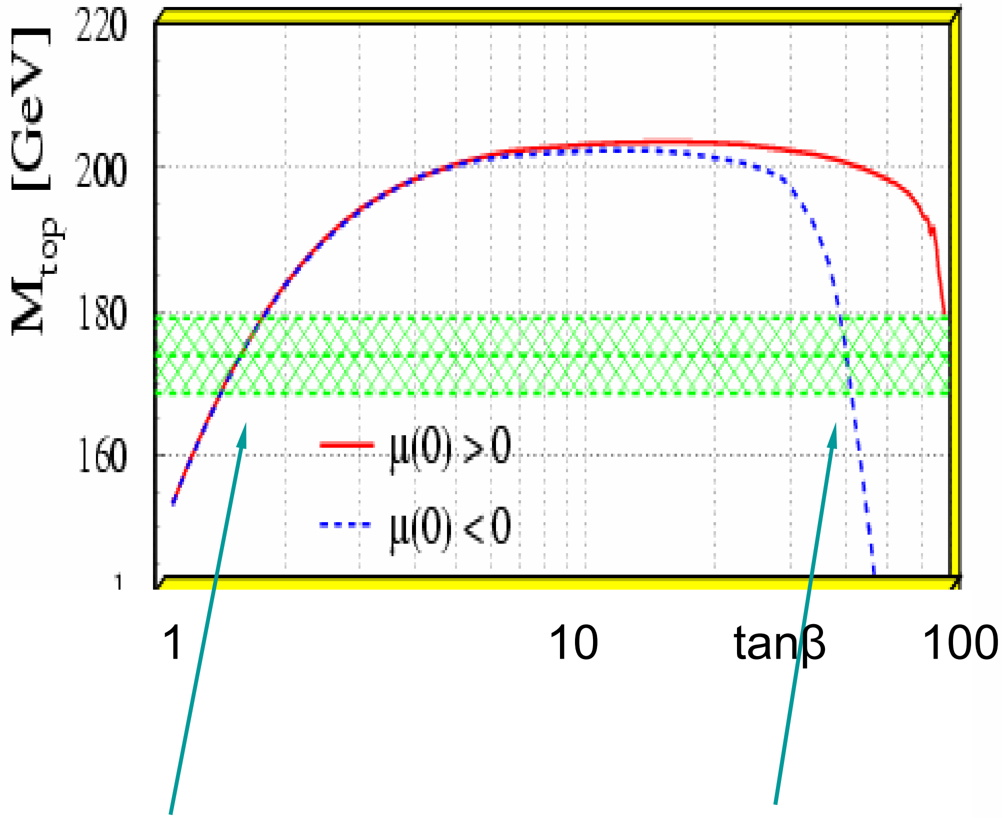


$M_{SUSY} \sim 1\ TeV$

Constrained MSSM

- Choice of constraints

  - Unification of the Yukawa coupling constants. Combination of b-quark and τ-lepton Yukawa couplings unification with the t-quark mass strongly constrains the tan β value.



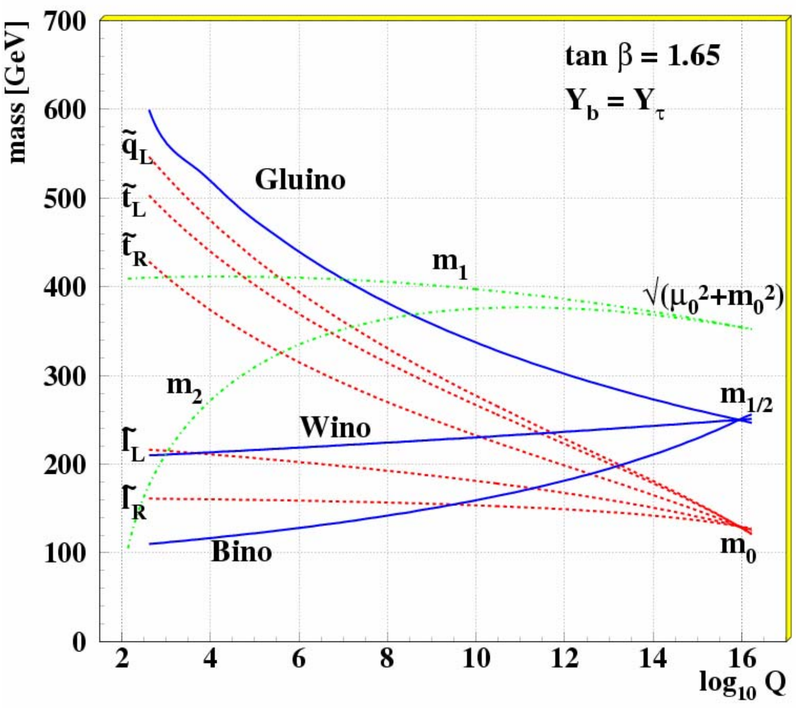
$$1 < \tan\beta < 3 \qquad 50 < \tan\beta < 70$$

- Small tan β cenario
- Large tan β cenario

Constrained MSSM

- Choice of constraints
  - Radiative electroweak symmetry breaking and Z-boson mass.

    It defines the μ parameter for given values of m_0. The sign of μ is undetermined.



$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2} \approx -m_{H_2}^2 - \frac{M_Z^2}{2}$$

Constrained MSSM

- Choice of constraints

- Let us fix the value of tan β
- Let us calculate the value of the μ parameter (up to the sign)
- Soft supersymmetry breaking parameter A_0 is irrelevant in most cases (A=0)

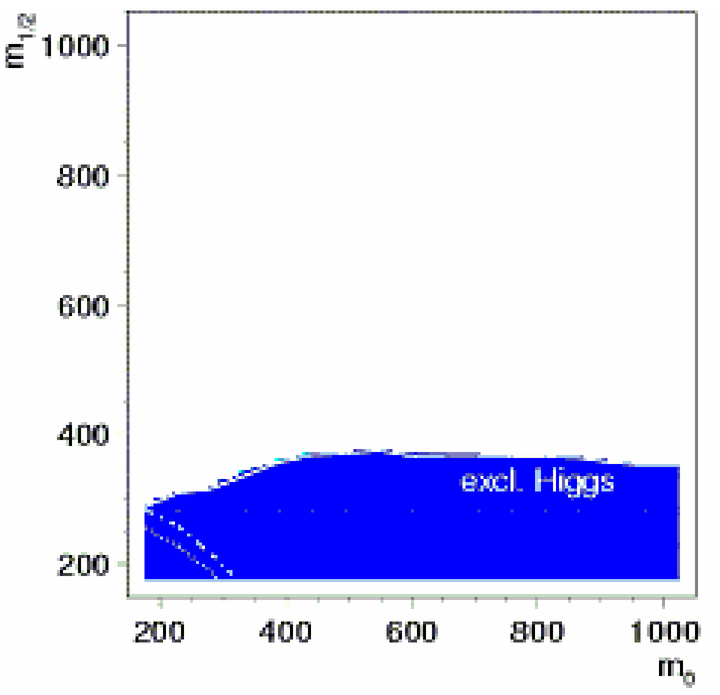
- We end up with only a pair of parameters

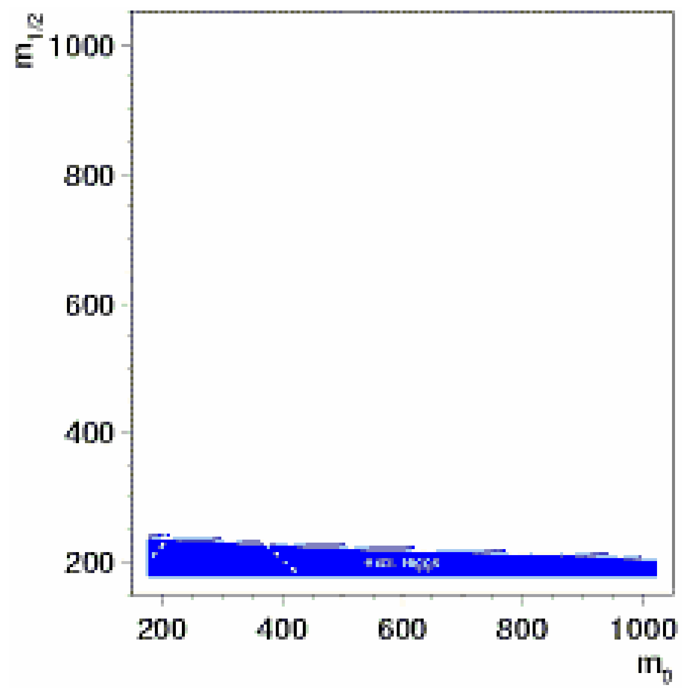
$$\mu,\ A,\ \underline{m_0,\ m_{1/2}},\ \tan\beta$$

- From now on we will use the $m_0 - m_{1/2}$ plane and look for allowed regions

Constrained MSSM

- Choice of constraints
  - Experimental bounds on the Higgs mass. The bound m_H > 114 GeV excludes tan β < 4 (from now on tan β = 35, 50).





Constrained MSSM

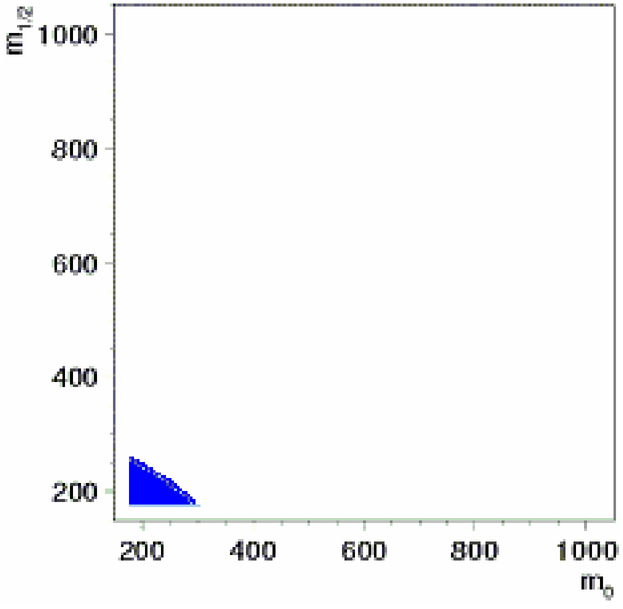
- Choice of constraints

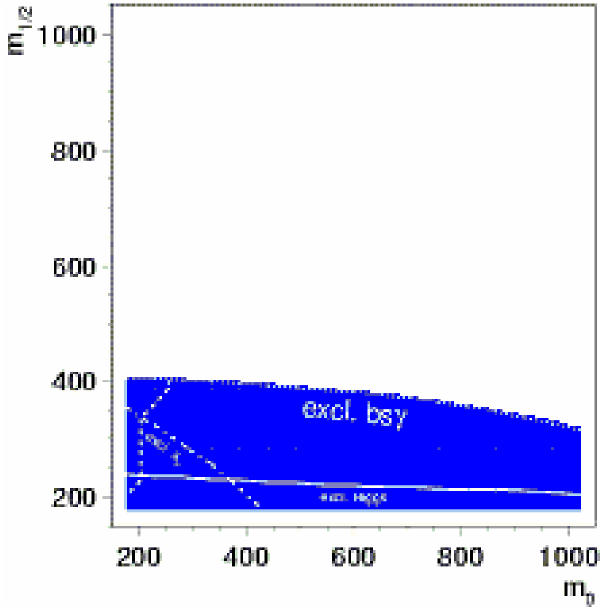
  - Precise measurements of rare decays branching ratios.
    Branching ratios of rare decays may be influenced by radiative corrections including superpartners in loops. Example: $b \to s\gamma$
    Experimental data exceed theoretical estimations within the Standard Model leaving a room for supersymmetry
  - Muon anomalous magnetic moment.
    Recent measurements point to a certain deviation from the Standard Model predictions. The gap can be filled with contribution of superpsrtners

Constrained MSSM

- Choice of constraints

$$b \to s\gamma$$

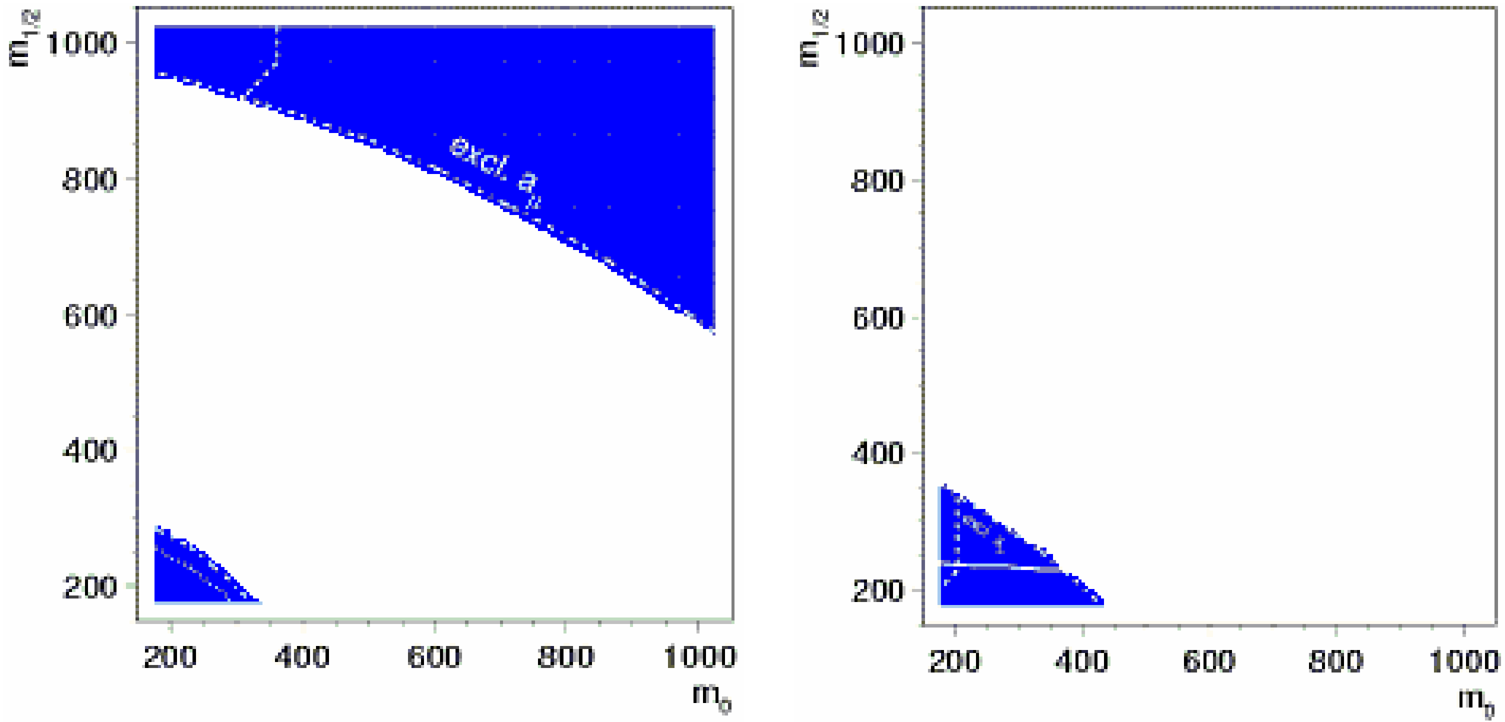




Constrained MSSM

- Choice of constraints

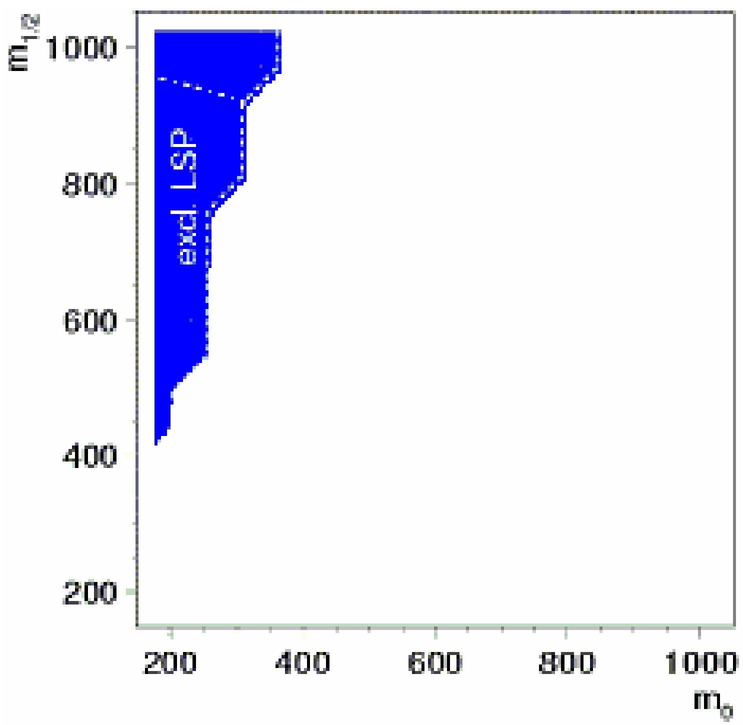
Muon anomalous magnetic moment requires positive μ

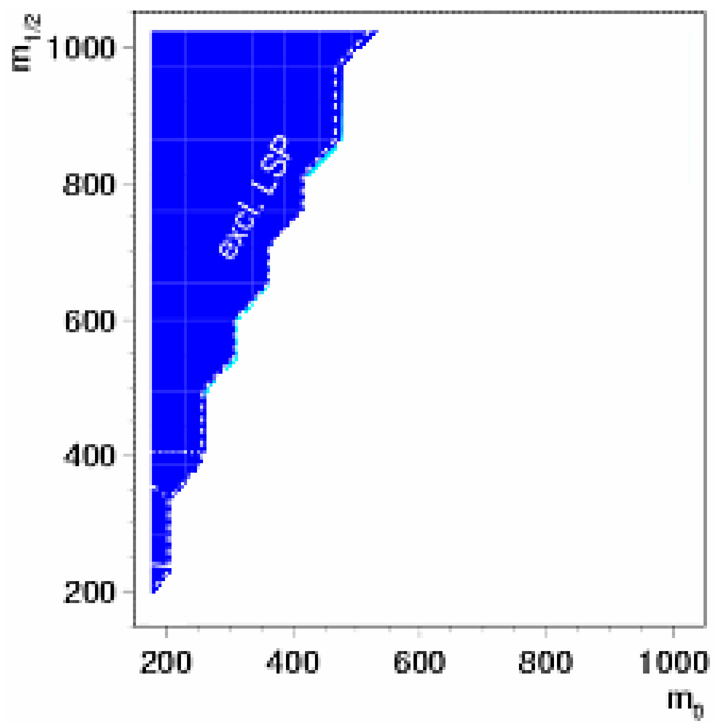


Constrained MSSM

- Choice of constraints

  - Neutrality of the lightest supersymmetric particle. Consequence of R-parity conservation





Constrained MSSM

- Choice of constraints

  - Experimental bounds on superpartner masses. Non-observation of superpartners constrains their masses from below (that is constrains the soft supersymmetry breaking parameters)

LEP2 SUSY particle search

- *pair slepton production:*

  $e^+e^- \to \tilde{\ell}^+_{L,R}\tilde{\ell}^-_{L,R} \to \ell^+\widetilde{Z}_1\ell^-\widetilde{Z}_1$

  $\Rightarrow m_{\tilde{e}} > 99.6\,\text{GeV},\ m_{\tilde{\mu}} > 94.6\,\text{GeV},\ m_{\tilde{\tau}} > 85.9\ \text{GeV}$

- *pair chargino production:*

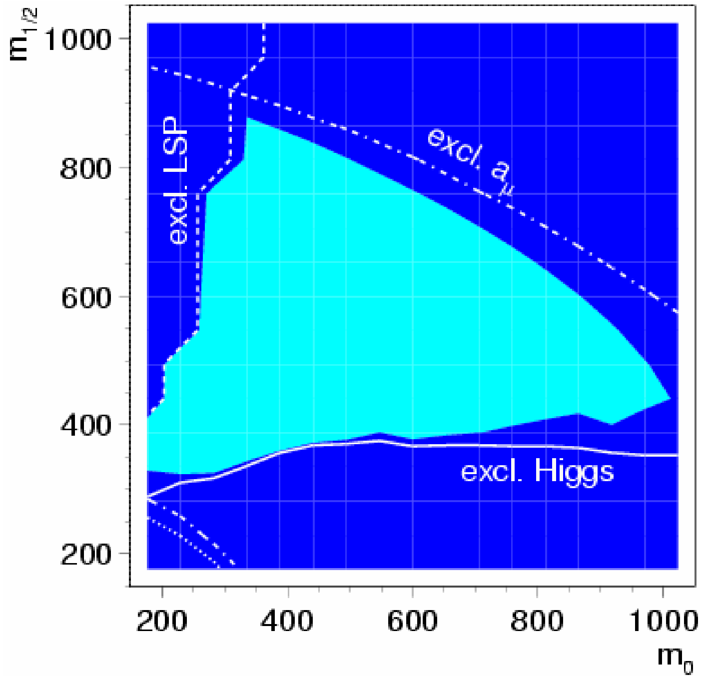
  $e^+e^- \to \widetilde{W}_1^+\widetilde{W}_1^-,\ \ \widetilde{W}_1 \to \widetilde{Z}_1\ell\nu(\widetilde{Z}_1 qq'),$

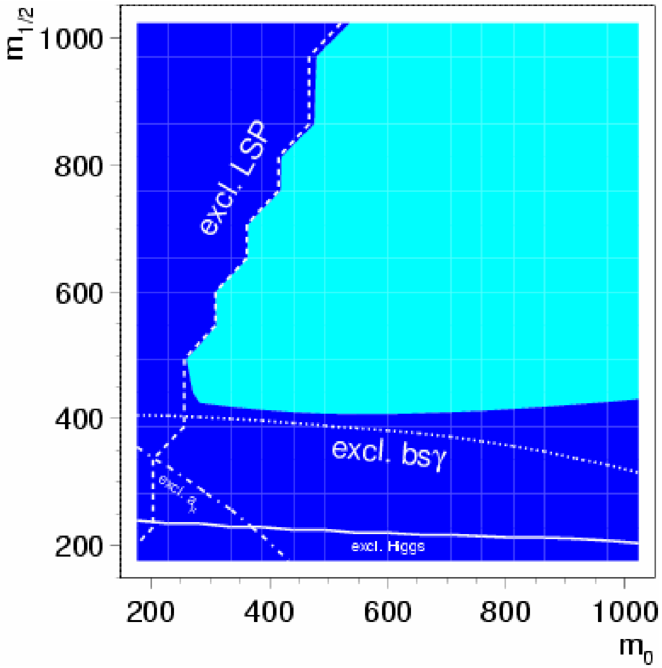
  $\Rightarrow m_{\widetilde{W}_1} \gtrsim 100\,\text{GeV}$

Constrained MSSM

- Choice of constraints

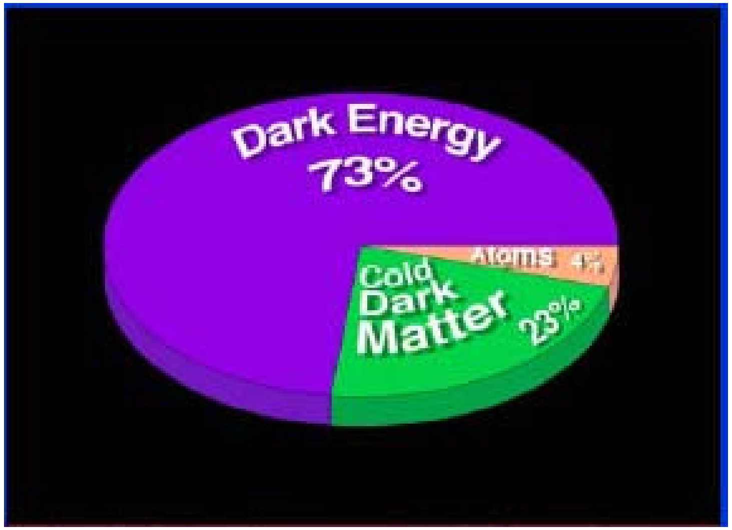
- Remarkable fact ia that all these constraints can be fulfilled simultaneously. As a result one can find optimal values of the parameters and allowed regions in the parameter space





Constrained MSSM

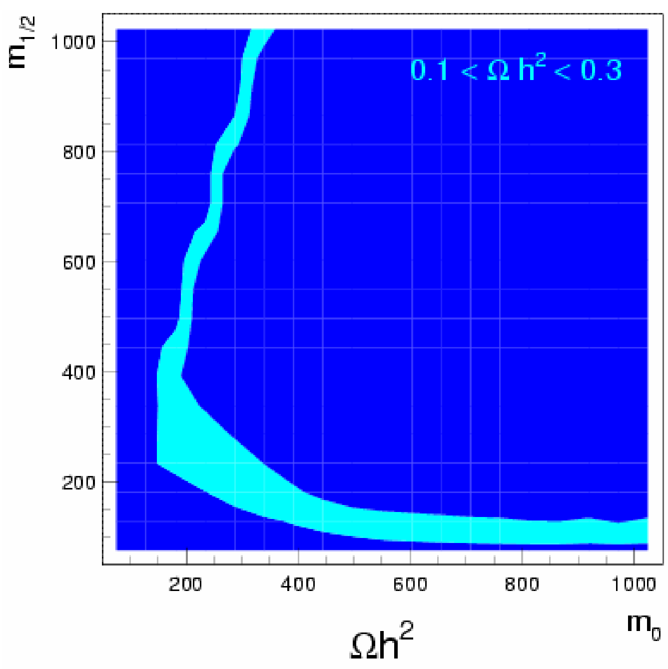
- Choice of constraints
  - Dark Matter in the Universe.
    MSSM has a good candidate
    for the WIMP – neutralino –
    a mixture of superpartners
    of photon, Z-boson and Higgses

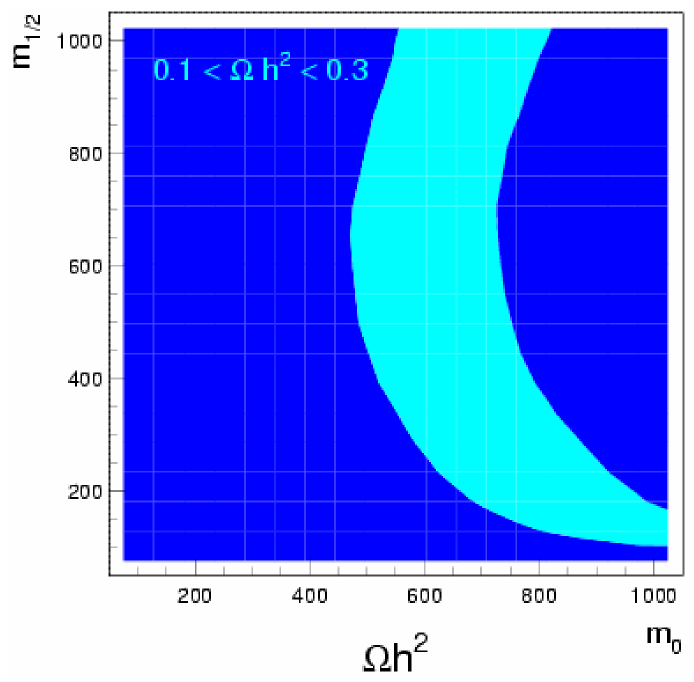


  - Neutral (no electric charge, no colour)
  - Weakly interacting (due to supersymmetry)
  - Stable (!) if R-parity is conserved
  - Heavy enough to account for cold non-baryonic dark matter

Constrained MSSM

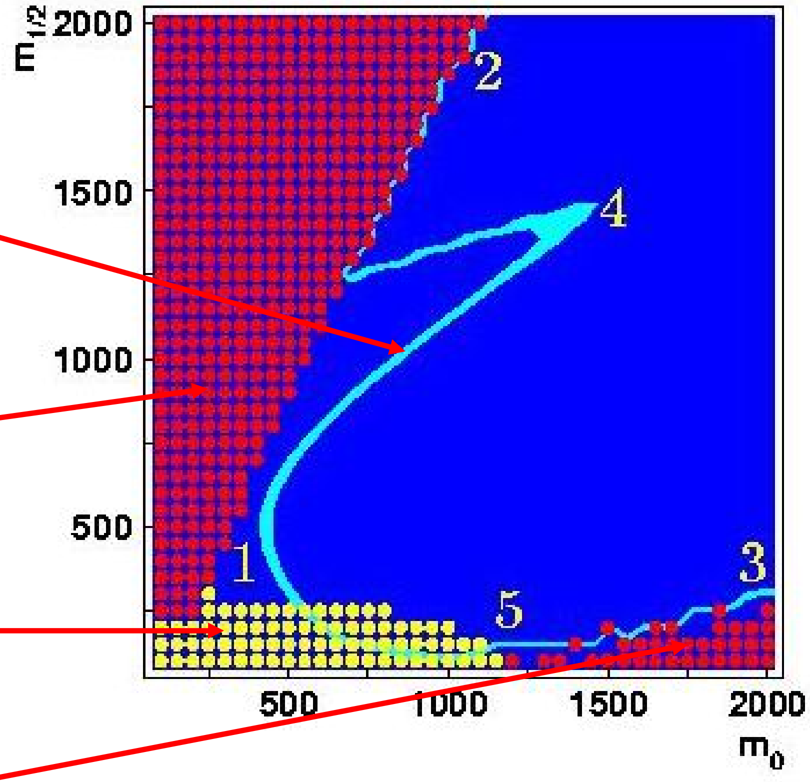
- Regions of the MSSM parameter space consistent with the dark matter constraint ($\Omega = 0.1 - 0.3$)





Constrained MSSM

- WMAP data leave only very small allowed region which give acceptable neutralino relic density
- Excluded by LSP
- Excluded by Higgs searches at LEP2
- Excluded by REWSB



m_0 – common scalar mass
$m_{1/2}$ – common gaugino mass

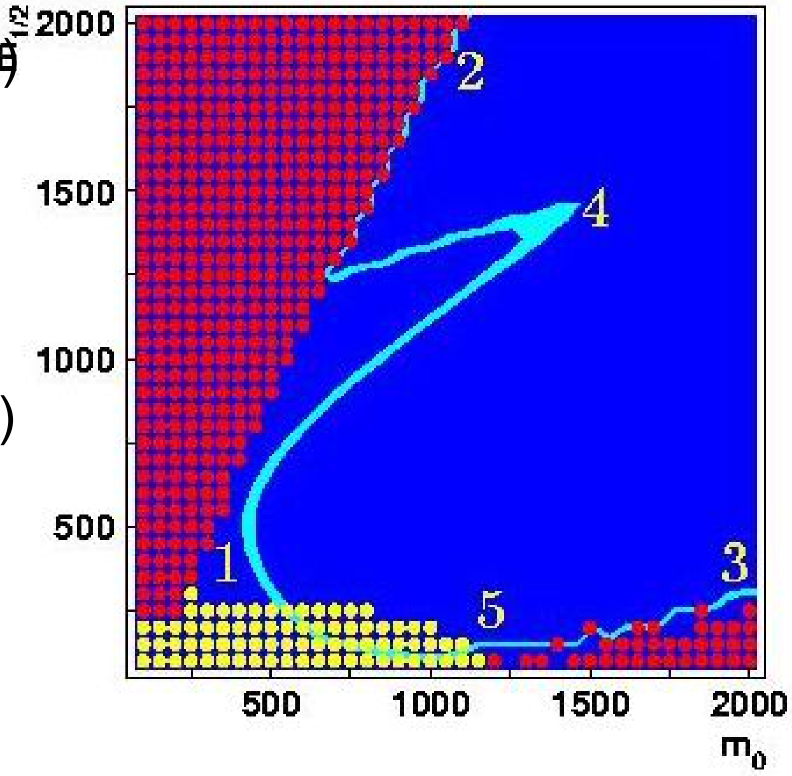
Constrained MSSM

- 1. Bulk region (low m_0 and low $m_{1/2}$)
- 2. Stau-coannihilation region

  (moderate m_0 but large $m_{1/2}$)
- 3. Focus point region

  (large m_0 and low to moderate $m_{1/2}$)
- 4. A-annihilation funnel region

  (the region reguires large tan β)
- 5. EGRET region (consistent with astrophysical data on diffuse gamma rays flux)



Constrained MSSM

- SUSY parameters and superparticle spectrum

Parameter	Value
m_0	1500 GeV
$m_{1/2}$	170 GeV
A_0	$0 \cdot m_0$
$\tan\beta$	52.2
sign μ	+
$\alpha_s(M_Z)$	0.122
$\alpha_{em}(M_Z)$	0.0078153697
$1/\alpha_{em}$	127.953
$\sin^2(\theta_W)_{\overline{MS}}$	0.2314
m_t	175 GeV
m_b	4.214 GeV

Particle	Mass [GeV]
$\tilde{\chi}^0_{1,2,3,4}$	64, 113, 194, 229
$\tilde{\chi}^\pm_{1,2}, \tilde{g}$	110, 230, 516
$\tilde{u}_{1,2} = \tilde{c}_{1,2}$	1519, 1523
$\tilde{d}_{1,2} = \tilde{s}_{1,2}$	1522, 1524
$\tilde{t}_{1,2}$	906, 1046
$\tilde{b}_{1,2}$	1039, 1152
$\tilde{e}_{1,2} = \tilde{\mu}_{1,2}$	1497, 1499
$\tilde{\tau}_{1,2}$	1035, 1288
$\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$	1495, 1495, 1286
$h, H, A, H^\pm$	115, 372, 372, 383

Observable	Value
$Br(b \to X_s\gamma)$	$3.02 \cdot 10^{-4}$
Δa_μ	$1.07 \cdot 10^{-9}$
Ωh^2	0.117

Sparticle physics

- Supersymmetry is the most popular idea beyond the Standard Model
- The new physics is expected at the TeV scale
- If we are right, the new discoveries are waiting for, and the table of fundamental particles has to be updated