Cosmology

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Outline of Lectures

Lecture 1: Basics of Cosmology
  - Expanding Universe
  - Warm Universe
  - Dynamics of expansion
  - Omega’s
  - Sample solutions
  - Big Bang Nucleosynthesis

Lecture 2: Dark matter

Lecture 3: Baryon asymmetry

Lecture 4: What do we know and how do we know? What are we going to learn?
Units and conventions

- $\hbar = c = k_B = 1$

- $1 \text{ Mpc} = 3 \cdot 10^6 \text{ light yrs} = 3 \cdot 10^{24} \text{ cm}$

  Our place in Galaxy: 8 kpc from Galactic center

  Nearest galaxy: Andromeda, 0.8 Mpc

  Clusters of galaxies: several Mpc

  Visible Universe: 15 Gpc
Basics of Cosmology

The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean. All this is encoded in space-time metric

\[ ds^2 = dt^2 - a^2(t)dx^2 \]

\( x \): comoving coordinates, label distant galaxies.

\( a(t)dx \): physical distances.

\( a(t) \): scale factor, grows in time; \( a_0 \): present value (matter of convention)

**NB:** Three homogeneous isotropic spaces:
- Euclidean 3-space
- 3-sphere
- 3-hyperboloid

Observationally our 3-space is Euclidean to a very good approximation. Sum of angles of a triangle = 180 degrees even for triangles of size 10 Gpc
**Expanding Universe**

- Scale factor $a(t)$ grows in time

$$z(t) = \frac{\text{photon wavelength today}}{\text{wavelength at emission time } t} - 1 = \frac{a_0}{a(t)} - 1 : \text{ redshift}$$

$$H(t) = \frac{\dot{a}}{a} : \text{ Hubble parameter, expansion rate}$$

- Present value

$$H_0 = [70 \pm 2] \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9 \text{ yrs})^{-1}$$

**NB:** Units: naive interpretation: $z(t) = v(r = t)$, velocity of emitting galaxy

- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$
Hubble diagram for SNeIa

\[ \text{mag} = 5 \log_{10} r + \text{const} \]
Since the Universe expands,

- Early Universe: $a(t)$ much smaller than today; matter density much higher than today.
Warm Universe

Manifestation: Cosmic Microwave Background, CMB, thermal gas of photons.
CMB photons do not interact with matter today.
Matter and photons were in thermal equilibrium in early Universe.

CMB temperature today

\[ T_0 = 2.725 \text{ K} \]

It was denser and warmer at early times.
Frequency of CMB photon:

\[ \omega(t) = \frac{a_0}{a(t)} \omega_0 \]

Hence

\[ T(t) = \frac{a_0}{a(t)} T_0 \]
CMB spectrum

\[ T = 2.725 \text{ K} \]
Present number density of photons

\[ n_\gamma = \#T^3 = 410 \frac{1}{\text{cm}^3} \]

Present entropy density

\[ s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\text{cm}^3} \]

In early Universe

\[ s = \frac{2\pi^2}{45} g_* T^3 \]

\( g_* \): number of degrees of freedom with \( m \ll T \).

Entropy density scales exactly as \( a^{-3} \)

Temperature scales approximately as \( a^{-1} \).
Recombination:

- Early Universe was hot \( \implies \text{matter was in plasma phase} \)
- Mostly electrons and protons (with 25\% admixture of \( ^4\text{He} \) nuclei)
- Plasma opaque for photons. \( e, p, \gamma \) in thermal equilibrium

- At temperature \( T = 0.27 \text{ eV} \approx 3000 \text{ K} \) electrons and protons combine into hydrogen atoms
  \( \implies \text{Universe became transparent to photons} \)

Since then CMB photons travel freely, and just get redshifted

- Redshift at recombination:
  \[
  z_{rec} = \frac{T_{rec}}{T_0} \approx 1100
  \]

- Lifetime of the Universe at recombination: \( t \approx 300 \text{ 000 yrs} \)
  (13.6 billion yrs today)
Plasma – gas transition

300 тысяч лет
T=3000 градусов
Dynamics of expansion

- **Friedmann equation**: expansion rate of the Universe vs **total** energy density $\rho$:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2} \rho$$

$(M_{Pl} = G^{-1/2} = 10^{19}$ GeV), no spatial curvature.

- **Cosmological horizon**: distance that signals can travel from Big Bang to time $t$,

$$l(t) \sim H(t)^{-1}$$

Today $l_0 \approx 15$ Gpc $= 4.5 \cdot 10^{28}$ cm

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi} H_0^2 = 5 \cdot 10^{-6} \frac{\text{GeV}}{\text{cm}^3}$$
Energy balance in the present Universe

- 70–75% — dark energy
- 25–20% — dark matter
- 4.5% — ordinary matter
- 0.5% — stars
- 0.3–1% — neutrino
Different components behave differently in time:

- **Matter** (dark matter + baryons):
  \[ \rho_M(t) \propto n_M(t) \propto \frac{1}{a^3(t)} \]

- **Radiation** (photons + neutrinos at early times):
  \[ T(t) \propto 1/a(t), \]

  \[ \rho_{\text{rad}}(t) \propto n_{\gamma}(t) T(t) \propto \frac{1}{a^4(t)} \]

- **Dark energy** \( \Lambda \):
  \[ \rho_{\Lambda} \approx \text{const} \]
Omega’s

\[ \Omega_i = \frac{\text{Present energy density of } i^{\text{th}} \text{ fraction}}{\text{Total present energy density}} = \frac{\rho_i}{\rho_{\text{tot}}(\text{today})} \]

In case of massless neutrinos (sufficient for us):

\[ \Omega_{DM} = 0.2 - 0.25 \]
\[ \Omega_B = 0.045 \]
\[ \Omega_M = \Omega_{DM} + \Omega_B = 0.3 - 0.25 \]
\[ \Omega_{rad} = \Omega_\gamma + \Omega_\nu \approx 10^{-4} \]
\[ \Omega_\Lambda = 0.7 - 0.75 \]

\[ \sum_i \Omega_i = 1 \]
Friedmann equation again:

$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}(\rho_M + \rho_{rad} + \rho_\Lambda)$$

$$= H_0^2 \left[ \Omega_M \left( \frac{a_0}{a(t)} \right)^3 + \Omega_{rad} \left( \frac{a_0}{a(t)} \right)^4 + \Omega_\Lambda \right]$$

Radiation domination $\implies$ Matter domination $\implies$ $\Lambda$-domination
Sample solutions

- **Radiation domination** (early Universe)

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3M_{Pl}} \rho_{rad} = \text{const} \cdot \frac{1}{a^4}
\]

\[\implies a(t) = \text{const} \cdot \sqrt{t} ; \quad H(t) = \frac{1}{2t}\]

- **Singularity, \( t = 0 \):**

\[\rho_{rad} \propto \left( \frac{\dot{a}}{a} \right)^2 \propto \frac{1}{t^2}\]

- **Decelerated expansion:**

\[\ddot{a} \propto -\frac{1}{t^{3/2}}\]
Cosmological horizon increases with time

- Consider times $t_1$ and $t_2 > t_1$.

Cosmological horizon at time $t_1$: distance which light travels from $t = 0$ to $t_1$,

$$l_1(t_1) = t_1 \quad \text{(omitting a number of order 1)}$$

- Universe expands from $t_1$ to $t_2 \implies$ size of this region at time $t_2$:

$$l_1(t_2) = l_1(t_1) \cdot \frac{a(t_2)}{a(t_1)} = t_1 \frac{\sqrt{t_2}}{\sqrt{t_1}} = \sqrt{t_1 t_2}$$

- The point is that the horizon at time $t_2$ is larger,

$$l_2(t_2) = t_2 > \sqrt{t_1 t_2} = l_1(t_2)$$

More and more Universe becomes visible.

Similar properties of the Universe filled with non-relativistic matter
In other words, we see many (∼1000) regions which were causally disconnected at recombination.

How come these regions are all the same?  
Plausible answer: inflation
\( \Lambda \)-domination, \( \rho_\Lambda = \text{const} \)

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \rho_\Lambda = \text{const}
\]

\[ \implies a(t) = \text{const} \cdot e^{ct} \]

\[ c = \sqrt{\frac{8\pi}{3M_{Pl}}} \rho_\Lambda. \]

\textbf{NB:} if only \( \Lambda \), No Big Bang singularity (cf. inflation)

\[ \text{\textbullet \ Accelerated expansion:} \]

\[ \ddot{a} \propto +e^{ct} \]

Observations: expansion \textit{accelerates}.

Our Universe enters stage of \( \Lambda \)-domination
Primordial nucleosynthesis (BBN)

Temperature $10^{10} \rightarrow 10^9$ K = 1 MeV $\rightarrow$ 50 keV, age of the Universe $1 \rightarrow 200$ s.

\[ p + n \rightarrow D + \gamma \]
\[ D + D \rightarrow ^3He + n , \quad D + D \rightarrow T + p \]
\[ ^3He + D \rightarrow ^4He + p , \text{ etc.} \]

Parameters:
- baryon $(p, n)$ density i.e., baryon-photon ratio $n_B/n_\gamma = \eta$;
- number of neutrino species (3 in Standard Model)

Comparison between theory and observations of element abundances $\Rightarrow$ composition and evolution of the Universe at $t = 1 \text{ s} \rightarrow 3 \text{ min.}$
$\Rightarrow$ baryon density at that time and at present
\[ \eta_{10} = \eta \cdot 10^{-10}. \]

Consistent with CMB determination of \( \eta \).
Outline of Lecture 2

- Dark matter: astrophysical evidence
- Dark matter: cosmological evidence
- Warm vs. cold dark matter
- WIMPs
- Gravitinos as warm dark matter candidates
Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
- Velocity curves of galaxies
- Velocities of galaxies in clusters
  - Original Zwicky’s argument, 1930’s

\[ v^2 = G \frac{M(r)}{r} \]

- Temperature of gas in X-ray clusters of galaxies
- Gravitational lensing of clusters
- Etc.
Rotation curves

M33 rotation curve

observed

expected from luminous disk
Gravitational lensing
Outcome

\[ \Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.25 - 0.3 \]

Assuming mass-to-light ratio everywhere the same as in clusters
NB: in clusters sit 10 % of galaxies

Nucleosynthesis, CMB:

\[ \Omega_B = 0.045 \]

The rest is non-baryonic, \( \Omega_{DM} \approx 0.2 \).

Physical parameter: mass-to-entropy ratio. Stays constant in time. Its value

\[
\left( \frac{\rho_{DM}}{s} \right)_0 = \frac{\Omega_{DM} \rho_c}{s_0} = \frac{0.2 \cdot 5 \cdot 10^{-6} \text{ GeV cm}^{-3}}{3000 \text{ cm}^{-3}} = 3 \cdot 10^{-10} \text{ GeV}
\]
Cosmological evidence: growth of structure

**CMB anisotropies:** baryon density perturbations at recombination $\approx$ photon last scattering, $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left( \frac{\delta \rho_B}{\rho_B} \right)_{z=1100} \simeq \left( \frac{\delta T}{T} \right)_{CMB} = (\text{a few}) \cdot 10^{-5}$$

In matter dominated Universe, matter perturbations grow as

$$\frac{\delta \rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only
If not for dark matter,

$$\left( \frac{\delta \rho}{\rho} \right)_{today} = 1100 \times (\text{a few}) \cdot 10^{-5} = (\text{a few}) \cdot 10^{-2}$$

No galaxies, no stars...
Perturbations in dark matter start to grow much earlier
(already at radiation-dominated stage)
Growth of perturbations (linear regime)

Radiation domination \hspace{2cm} Matter domination \hspace{2cm} \Lambda\,\text{domination}

\begin{align*}
\delta_{\gamma} & \hspace{2cm} \delta_{DM} \\
\delta_{B} & \hspace{2cm} \Phi
\end{align*}

\begin{align*}
t_{eq} & \hspace{2cm} t_{rec} \hspace{2cm} t_{\Lambda} \hspace{2cm} t
\end{align*}
NB: Need dark matter particles non-relativistic early on. Neutrinos are not considerable part of dark matter

**UNKNOWN DARK MATTER PARTICLES ARE CRUCIAL FOR OUR EXISTENCE**

Cold dark matter, CDM

\[ m_{DM} > \text{a few} \cdot 10 \text{ keV} \]

Warm dark matter

\[ m_{DM} \simeq 1 - 10 \text{ keV} \]
Warm dark matter

- Motion of dark matter particles supresses formation of small structures = dwarf galaxies

\[ M_{\text{dwarf}} \sim (10^8 - 10^9) M_\odot \]

(normal galaxies: \( M \sim (10^{10} - 10^{12}) M_\odot \))

Suppression of dwarfs may or may not be necessary for agreement between numerical simulations of structure formation and observations

- Controversy
Digression: expansion of the early Universe

- Friedmann equation:

\[
\left( \frac{\dot{a}}{a} \right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}} \rho
\]

- Early epoch (radiation dominated): Stefan–Boltzmann

\[
\rho = \frac{\pi^2}{30} g_* T^4
\]

\(g_*\): number of relativistic degrees of freedom (about 100 in SM at \(T \sim 100\) GeV). Hence

\[
H(T) = \frac{T^2}{M_{Pl}^*}
\]

with \(M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}\) GeV at \(T \sim 100\) GeV

Example. Time scale at \(T = 100\) GeV:

\[
H^{-1}(T) = M_{Pl}^*/T^2 \sim 10^{14}\text{ GeV}^{-1} \sim 10^{-10}\text{ s}
\]
Warm dark matter

- Decouples when relativistic, $T \gg m$.
- Remains relativistic until $T \sim m$ (assuming thermal distribution). Does not feel gravitational potential before that.
- Perturbations of wavelengths shorter than horizon size at that time get smeared out $\implies$ small size objects do not form ("free streaming" $\approx$ Landau damping)
- Horizon size at $T \sim m$

$$l(T) = H^{-1}(T \sim m) = \frac{M^*_{Pl}}{T^2} = \frac{M^*_{Pl}}{m^2}$$

Present size of this region

$$l(t_0) = \frac{a_0}{a(T)} l(T) = \frac{T}{T_0} l(T) = \frac{M_{Pl}}{mT_0}$$

(modulo $g_*$ factors).
Objects of size smaller than $l(t_0)$ are less abundant
Initial size of dwarf galaxy \( l_{dwarf} \sim 100 \text{ kpc} \sim 3 \cdot 10^{23} \text{ cm} \)

Require

\[
l(t_0) \sim \frac{M_{Pl}}{mT_0} \sim l_{dwarf}
\]

\[\implies \text{obtain mass of DM particle}\]

\[
m \sim \frac{M_{Pl}}{T_0 l_{dwarf}} \sim 3 \text{ keV}
\]

\((M_{Pl} = 10^{19} \text{ GeV}, T_0^{-1} = 0.1 \text{ cm}).\)

Particles of masses in keV range
are good warm dark matter candidates
WIMPs: cold dark matter candidates

Simple but very suggestive scenario

- Assume there is a new heavy stable particle $X$
- Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q}, \text{etc}$$

- Mass: $M_X$, annihilation cross section at non-relativistic velocity $v$: $\sigma(v)$
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density
Number density of $X$-particles in equilibrium at $T < M_X$:
Maxwell–Boltzmann

$$n_X = g_X \left( \frac{M_X T}{2\pi} \right)^{3/2} e^{-\frac{M_X}{T}}$$

Mean free time wrt annihilation:

$$\tau_{ann} \equiv \Gamma_{ann}^{-1} = \frac{1}{n_X \langle \sigma v \rangle}$$

Freeze-out: $\Gamma_{ann}(T_f) \sim H(T_f) \implies n_X(T_f) \langle \sigma v \rangle \sim T_f^2 / M_{Pl}^*$:

$$g_X \langle \sigma v \rangle \left( \frac{M_X T_f}{2\pi} \right)^{3/2} e^{-\frac{M_X}{T_f}} = \frac{T_f^2}{M_{Pl}^*} \implies T_f \sim \frac{M_X}{\log(M_X M_{Pl}^* \langle \sigma v \rangle)}$$

NB: large log $\iff T_f \sim M_X / 30$

Define $\langle \sigma v \rangle \equiv \sigma_0$ (constant for s-wave annihilation)
Number density at freeze-out

\[ n_X \sigma_0 = \Gamma_{\text{ann}} = H = \frac{T_f^2}{M_{Pl}^*} \implies n_X(T_f) = \frac{T_f^2}{\sigma_0 M_{Pl}^*} \]

Number-to-entropy ratio at freeze-out and later on

\[
\frac{n_X(T_f)}{s(T_f)} = \# \frac{n_X(T_f)}{g^* T_f^3} = \# \frac{\log(M_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g^* M_{Pl}^*}
\]

where \( \# = 45/(2\pi^2) \).

Mass-to-entropy ratio

\[
\frac{M_X n_X}{s} = \# \frac{\log(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g^*(T_f) M_{Pl}}}
\]

Most relevant parameter: annihilation cross section \( \sigma_0 \equiv \langle \sigma v \rangle \) at freeze-out
\[ \frac{M_X n_X}{s} = \# \frac{\log(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}} \]

Correct value, mass-to-entropy = \(3 \cdot 10^{-10}\) GeV, at

\[ \sigma_0 \equiv \langle \sigma v \rangle = (1 \div 2) \cdot 10^{-36} \text{ cm}^2 \]

Weak scale cross section.

Gravitational physics and EW scale physics combine into

\[ \text{mass-to-entropy} \approx \frac{1}{M_{Pl}} \left( \frac{\text{TeV}}{\alpha_W} \right)^2 \approx 10^{-10} \text{ GeV} \]

Mass \(M_X\) should not be much higher than 100 GeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates
SUSY: neutralinos, $X = \chi$

But situation is rather tense already: annihilation cross section is often too low

Important suppression factor: $\langle \sigma v \rangle \propto v \propto \sqrt{T/M_\chi}$ because of $p$-wave annihilation in case $\chi \chi \rightarrow Z^* \rightarrow f \bar{f}$:

Relativistic $f \bar{f} \implies$ total angular momentum $J = 1$

$\chi \chi$: identical fermions $\implies L = 0$, parallel spins impossible $\implies p$-wave
mSUGRA at fairly low \( \tan \beta \)

\[ \tan \beta = 10, \ \mu > 0 \]

\[ m_h = 114 \text{ GeV} \]
Larger $\tan \beta$ is better

$$m_0 \, (\text{GeV})$$

$$m_{1/2} \, (\text{GeV})$$

$\tan \beta = 50, \, \mu > 0$

$m_h = 114 \, \text{GeV}$
Alternatives to WIMPS: numerous

- CDM
- Axions
- Wimpzillas: Very heavy particles produced at the end of inflation
- Sneutrino, axino, .......

- Warm dark matter
- Gravitinos
- Sterile neutrinos, .......

- In some cases discovery at LHC possible, e.g. gravitinos
- In any case, extension of Standard Model
When dark matter particles observed and studied:

A handle on the Universe at

\[ T = (\text{a few}) \cdot 10 \text{ GeV} \div (\text{a few}) \cdot 100 \text{ GeV} \]

\[ t = 10^{-11} \div 10^{-8} \text{ s} \]

cf. \( T = 1 \text{ MeV}, \ t = 1 \text{ s} \) at nucleosynthesis
Gravitinos

- Mass \( m_{3/2} \approx F / M_{Pl} \)
  \[ \sqrt{F} = \text{SUSY breaking scale.} \]
  \[ \implies \text{Gravitinos light for low SUSY breaking scale.} \]
  \[ \text{E.g. gauge mediation} \]

- Light gravitino = LSP \( \implies \) Stable

- Decay width of superpartners into gravitino + SM particles

\[ \Gamma_{\tilde{S}} \approx \frac{M_{\tilde{S}}^5}{F^2} \approx \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} \]

\( M_{\tilde{S}} = \text{mass of superpartner} \tilde{S} \)
Gravitino production in decays of superpartners

\[
\frac{d(n_{3/2}/s)}{dt} = \frac{n_{\tilde{S}}}{s} \Gamma_{\tilde{S}}
\]

\[n_{\tilde{S}}/s = \text{const} \sim g_*^{-1} \text{ for } T \gtrsim M_{\tilde{S}}, \text{ while } n_{\tilde{S}} \propto e^{-M_{\tilde{S}}/T} \text{ for } T \ll M_{\tilde{S}}\]

\[\Rightarrow \text{ production most efficient at } T \sim M_{\tilde{S}} \text{ (slow cosmological expansion with unsuppressed } n_{\tilde{S}})\]

\[
\frac{n_{3/2}}{s} \sim \frac{\Gamma_{\tilde{S}}}{g_* H(T \sim M_{\tilde{S}})} \sim \frac{M_{Pl}^*}{g_* M_{\tilde{S}}^2} \cdot \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2}
\]

Mass-to-entropy ratio

\[
\frac{m_{3/2} n_{3/2}}{s} \sim \frac{M_{\tilde{S}}^3}{m_{3/2}^3/2} \frac{1}{g_* M_{Pl}}
\]
\[
\frac{m_{3/2} n_{3/2}}{s} \simeq \sum_{\tilde{S}} \frac{M_{\tilde{S}}^3}{m_{3/2}} s \sqrt{\frac{1}{g_* M_{Pl}}}
\]

For \( m_{3/2} = \text{a few keV} \), mass-to-entropy = \( 3 \cdot 10^{-10} \) GeV

\[
M_{\tilde{S}} \simeq 100 \div 300 \text{ GeV}
\]

Need light superpartners

and low maximum temperature in the Universe, \( T_{\text{max}} \lesssim 1 \text{ TeV} \) to avoid overproduction in collisions of superpartners (and in decays of squarks and gluinos if they are heavy)

Rather contrived scenario, but generating warm dark matter is always contrived

NB: \( \Gamma_{NLSP} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} \implies c \tau_{NLSP} = \text{a few} \cdot \text{mm} \div \text{a few} \cdot 100 \text{ m} \)

for \( m_{3/2} = 1 \div 10 \text{ keV} \), \( M_{\tilde{S}} = 100 \div 300 \text{ GeV} \)

Longer lifetime for heavier gravitino (CDM candidate)
Outline of Lecture 3

- Baryon asymmetry, preliminaries
- Sakharov conditions
- Electroweak $B$-violation
- Electroweak transition
- What can make EW mechanism work
- Mechanism of electroweak $B$-violation
Baryon asymmetry of the Universe

There is matter and no antimatter in the present Universe.

Baryon-to-photon ratio, almost constant in time:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 6 \cdot 10^{-10}$$

Baryon-to-entropy, constant in time: $n_B/s = 0.9 \cdot 10^{-10}$

What’s the problem?

Early Universe ($T > 10^{12}$ K = 100 MeV):
creation and annihilation of quark-antiquark pairs $\Rightarrow$

$$n_q, n_{\bar{q}} \approx n_\gamma$$

Hence

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-9}$$

How was this excess generated in the course of the cosmological evolution?

Sakharov’67, Kuzmin’70
Sakharov conditions

To generate baryon asymmetry, three necessary conditions should be met at the same cosmological epoch:

- **B-violation**
- **C- and CP-violation**
- **Thermal inequilibrium**

NB. Reservation: \( L \)-violation with \( B \)-conservation at \( T \gg 100 \) GeV would do as well \( \Rightarrow \) Leptogenesis.
Can baryon asymmetry be due to electroweak physics?

Baryon number is violated in electroweak interactions.

Non-perturbative effect

Hint: triangle anomaly in baryonic current $B^\mu$:

$$\partial_\mu B^\mu = \left( \frac{1}{3} \right)_{B_q} \cdot 3_{\text{colors}} \cdot 3_{\text{generations}} \cdot \frac{g_W^2}{32\pi^2} \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

$F^a_{\mu\nu}$: $SU(2)_W$ field strength; $g_W$: $SU(2)_W$ coupling

Likewise, each leptonic current ($n = e, \mu, \tau$)

$$\partial_\mu L^\mu_n = \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$
Large field fluctuations, \( F_{\mu \nu}^a \propto g_w^{-1} \) may have

\[
Q \equiv \int d^3xdt \frac{g_w^2}{32\pi^2} \varepsilon^{\mu \nu \lambda \rho} F_{\mu \nu}^a F_{\lambda \rho}^a \neq 0
\]

Then

\[
B_{\text{fin}} - B_{\text{in}} = \int d^3xdt \partial_\mu B^\mu = 3Q
\]

Likewise

\[
L_{n, \text{fin}} - L_{n, \text{in}} = Q
\]

\( B \) is violated, \( B - L \) is not.
Need large field fluctuations. At zero temperature their rate is suppressed by

\[ e^{-\frac{16\pi^2}{g_W^2}} \sim 10^{-165} \]

High temperatures: large thermal fluctuations ("sphalerons").

\[ B \]-violation rapid as compared to cosmological expansion at

\[ \langle \phi \rangle_T < T \]

\( \langle \phi \rangle_T \): Higgs expectation value at temperature \( T \).

Possibility to generate baryon asymmetry at electroweak epoch, \( T_{EW} \sim 100 \text{ GeV} \)?

Obstacle: Universe expands slowly. Expansion time

\[ H^{-1} = \frac{M_{Pl}^*}{T_{EW}^2} \sim 10^{14} \text{ GeV}^{-1} \sim 10^{-10} \text{ s} \]

Too large to have deviations from thermal equilibrium?
The only chance: 1st order phase transition, highly inequilibrium process
Electroweak transition

Electroweak symmetry is restored at high temperatures

Just like superconducting state becomes normal at “high” $T$

Transition may in principle be 1st order

1st order phase transition occurs from supercooled state via spontaneous creation of bubbles of new (broken) phase in old (unbroken) phase.

Bubbles then expand at $v \sim 0.1c$

Beginning of transition: about one bubble per horizon

Bubbles born microscopic, $r \sim 10^{-16}$ cm, grow to macroscopic size, $r \sim 0.1H^{-1} \sim$ mm, before their walls collide

Boiling Universe, strongly out of equilibrium
$V_{eff}(\phi)$

1st order

2nd order
$\phi = 0$

$\phi \neq 0$
Baryon asymmetry may be generated in the course of phase transition, provided there is enough $C$- and $CP$-violation.

**Necessary condition:**

Baryon asymmetry generated during transition should not be washed out afterwards

$\implies$ $B$-violating processes must be switched off in broken phase

$\implies$ Just after transition

$$\langle \phi \rangle_T > T$$

Does this really happen?

Not in SM
Temperature-dependent effective potential, one loop

\[ V_{eff} = (-m^2 + \alpha T^2)|\phi|^2 - \frac{\beta}{3}T|\phi|^3 + \frac{\lambda}{4}|\phi|^4 \]

\[ \alpha = O(g^2), \quad \beta = O(g^3). \] Cubic term weird,

\[ -\frac{\beta}{3}T (\phi^\dagger \phi)^{3/2} \]

But crucial for 1st order phase transition. Obtains contributions from bosons only

\[ f_B = \frac{1}{e^{E/T} - 1} \approx \frac{T}{E} \equiv \frac{T}{\sqrt{p^2 + g^2|\phi|^2}} \approx \frac{T}{g|\phi|} \quad \text{at} \quad |p| \ll T, \quad g|\phi| \ll T \]

Bose enhancement \( \iff \) no analyticity in \( g^2|\phi|^2 \)
At phase transition \((-m^2 + \alpha T^2) = 0\),

\[ V_{eff} = -\frac{\beta}{3} T \phi^3 + \frac{\lambda}{4} \phi^4 \]

Hence

\[ \langle \phi \rangle_T = \frac{\beta}{\lambda} T = \# g_W T \]

Given the Higgs mass bound

\[ m_H = \sqrt{2\lambda v} > 114 \text{ GeV} \]

one finds \(\langle \phi \rangle_T < T\), asymmetry would be washed out even if generated

**Furtermore, in SM**

- No phase transition at all; smooth crossover
- Way too small \(CP\)-violation
What can make EW mechanism work?

- Extra bosons
  - Should interact strongly with Higgs(es)
  - Should be present in plasma at \( T \sim 100 \text{ GeV} \)
    \( \implies \) not much heavier than 300 GeV

E.g. light stop

- Plus extra source of \( CP \)-violation.
  Better in Higgs sector \( \implies \) Several Higgs fields

More generally, EW baryogenesis requires complex dynamics in EW symmetry breaking sector
at \( E \sim (\text{a few}) \cdot 100 \text{ GeV} \)

LHC’s FINAL WORD
Is EW the only appealing scenario?

By no means!

— Leptogenesis

— Something theorists never thought about

Why $\Omega_B \approx \Omega_{DM}$?
Are we on right track assuming that dark matter and baryon asymmetry were generated at the hot stage of cosmological evolution, not at post-inflationary reheating stage?
How can baryon number be not conserved without explicit $B$-violating terms in Lagrangian?

Consider massless fermions in background gauge field $\tilde{A}(x,t)$ (gauge $A_0 = 0$). Let $\tilde{A}(x,t)$ start from vacuum value and end up in vacuum. NB: This can be a fluctuation

Dirac equation

$$i \frac{\partial}{\partial t} \psi = i \gamma^0 \gamma (\partial - ig\tilde{A}) \psi = H_{\text{Dirac}}(t) \psi$$

Suppose for the moment that $\tilde{A}$ slowly varies in time. Then fermions sit on levels of instantaneous Hamiltonian,

$$H_{\text{Dirac}}(t) \psi_n = \omega_n(t) \psi_n$$

How do eigenvalues behave in time?
Dirac picture at $\tilde{A} = 0, \ t \to \pm \infty$
TIME EVOLUTION OF LEVELS
IN SPECIAL (TOPOLOGICAL) GAUGE FIELDS

Left-handed fermions
Right-handed

The case for QCD

\[ B = N_L + N_R \text{ is conserved, } Q^5 = N_L - N_R \text{ is not} \]
If only left-handed fermions interact with gauge field, then number of fermions is not conserved.

The case for $SU(2)_W$

Fermion number of every doublet changes in the same way.
NB: Non-Abelian gauge fields only (in 4 dimensions)

**QCD:** Violation of $Q^5$ is a fact.

In chiral limit $m_u, m_d, m_s \to 0$,
global symmetry is $SU(3)_L \times SU(3)_R \times U(1)_B$,
not symmetry of Lagrangian $SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$
Outline of Lecture 4

- Primordial perturbations and ways to measure them
  - Power spectrum
  - Understanding the CMB angular spectrum
  - Scalar tilt and tensor amplitude
  - How to observe primordial gravity waves
- Dark energy
  - How dark energy is “seen”
  - Who is dark energy?
- What are we going to learn?
- Neutrino masses from cosmology
What do we know about our Universe and what do we hope to learn?

How do we know and how do we hope to learn?

**NB:** Assume Einstein gravity in what follows. No need for modifying it, except, maybe, at $l, t \sim H_0^{-1}$ (as an alternative to dark energy)

**Characteristics of the Universe**

- Composition + spatial curvature
  - Baryons, neutrinos, dark matter, dark energy, something even more exotic; spatial curvature
- Properties of primordial perturbations
The Universe is not exactly homogeneous

Deep surveys of galaxies and quasars:
  more than $10^6$ objects $\implies$ map of the Universe
  out to distances $7000\ \text{Mpc} = 21 \cdot 10^9\ \text{light yrs}$

CMB anisotropies

Perturbations existed already at very early epoch.
At that time they had very small amplitude.
Then they grew and finally developed into structures.

Classification of primordial perturbations

- **Scalar** (helicity 0) $\implies$ Perturbations in energy density and
  associated gravitational potentials
  exist
- **Vector** (helicity 1) $\implies$ Local rotational motion
  not observed, not expected
- **Tensor** (helicity 2) $\implies$ Primordial gravitational waves
  not observed,
predicted by a class of inflationary models
Las Campanas, mid-90's
Galaxies

Quasars

SDSS, recent
\[ T = 2.725^\circ K, \quad \frac{\delta T}{T} \sim 10^{-5} \]
Primordial scalar perturbations

Gaussian random field

\[ \delta(x,t) = \frac{\delta \rho(x,t)}{\bar{\rho}(t)} \]

(modulo technicalities)

Power spectrum

\[ \delta(x) = \int dke^{ikx} \delta(k) \]

\[ \langle \delta(k)\delta^*(k') \rangle = \frac{P(k)}{(2\pi)^3} \delta(k - k') \]

NB: To compare the results, one usually converts \( P(k,t) \) to recombination (last scattering) epoch
$h \approx 0.7$, present Hubble parameter in units \((\text{km/s})/\text{Mpc}\)
Gaussian random field $\delta(k)$:
Correlators obey Wick’s theorem,

$$
\langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = 0
$$

$$
\langle \delta(k_1)\delta(k_2)\delta(k_3)\delta(k_4) \rangle = \langle \delta(k_1)\delta(k_2) \rangle \cdot \langle \delta(k_3)\delta(k_4) \rangle

+ \text{ permutations of momenta}
$$

$\langle \delta(k)\delta^*(k') \rangle$ means averaging over ensemble of Universes. Realization in our Universe is intrinsically unpredictable.

Hint on the origin: enhanced vacuum fluctuations of free quantum field

Free quantum field

$$
\phi(x,t) = \int d^3k e^{-ikx} \left( f_k^{(+)}(t)a_k^\dagger + e^{ikx}f_k^{(-)}(t)a_k \right)
$$

In vacuo $f_k^{(\pm)}(t) = e^{\pm i\omega_k t}$

Enhanced perturbations: large $f_k^{(\pm)}$. But in any case, Wick’s theorem valid
Scalar perturbations:

- **Adiabatic mode:** $\delta \rho, \delta T \neq 0$, but chemical composition same everywhere,

$$\delta \left( \frac{\rho_B}{s} \right) = \delta \left( \frac{\rho_{DM}}{s} \right) = 0$$

This must be the only mode if dark matter and baryon asymmetry were generated at hot stage

Otherwise

- **Isocurvature (entropy) modes:** $\delta T = 0 \implies \delta \rho \approx 0$, but

$$\delta \left( \frac{\rho_B}{s} \right) \neq 0 \quad \text{(Baryon iso)} \quad \text{or} \quad \delta \left( \frac{\rho_{DM}}{s} \right) \neq 0 \quad \text{(DM iso)}$$

**Can one tell?**
Understanding CMB anisotropy

Fourier decomposition of temperature fluctuations:

$$\frac{\delta T}{T}(\theta, \varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \varphi)$$

$a_{lm}$: independent Gaussian random variables, $\langle a_{lm} a_{l'm'}^* \rangle \propto \delta_{ll'} \delta_{mm'}$

$\langle a_{lm}^* a_{lm} \rangle = C_l$ are measured; usually shown $D_l = \frac{l(l+1)}{2\pi} C_l$

larger $l \iff$ smaller angular scales, shorter wavelengths

NB: One Universe, one realization of an ensemble $\Rightarrow$ cosmic variance $\Delta C_l / C_l \simeq 1 / \sqrt{2l}$

Physics:

- Primordial perturbations, built in before hot stage
- Development of sound waves in cosmic plasma from early hot stage to recombination $\Rightarrow$ composition of cosmic plasma
- Propagation of photons after recombination $\Rightarrow$ expansion history of the Universe
CMB anisotropy spectrum

\[ u(l+1)C_l / 2\pi \] [\mu K^2]

Angular scale

Multipole moment \( \ell \)
Perturbations in baryon-photon plasma $\iff$ sound waves. Comoving momentum $k$ conserved, physical momentum $k/a(t)$ gets redshifted. 

- Mode of comoving momentum $k$ oscillates as

$$\frac{\delta \rho}{\rho}(k,t) \propto \cos \left( \int_0^t dt \frac{v_s k}{a(t)} \right)$$

$v_s =$ sound velocity.

**NB:** Phase of oscillations fixed!

Early times, $k/a \ll H$: one mode is constant, another rapidly decays away.

- Perturbations in DM do not oscillate.

No pressure — no oscillations.
Effects on CMB:

All at last scattering

- Temperature perturbations, $\delta T \propto \delta \rho_{\gamma} \iff$ baryon-photon component
- Gravitational potential $\iff$ dark matter mostly
- Doppler effect (somewhat suppressed) $\iff$ baryon-photon component

Adiabatic mode: baryons, photons and dark matter work together

CDM iso: No oscillations in time — no oscillations in $l$
   (oversimplified)
   Same for baryon iso
Observations consistent with purely adiabatic

Isocurvature $\lesssim 10\%$

This does favor generation of baryon asymmetry and dark matter at hot stage.

BUT

Even small admixture of isocurvature mode(s) would make a big difference

Watch out Planck!

Launched May 2009
CMB angular spectrum encodes a lot of information

- **Positions of peaks**

\[
\frac{\delta \rho}{\rho}(k, t_{rec}) \propto \cos \left( \int_0^{t_{rec}} dt \frac{v_s k}{a(t_{rec})} \right)
\]

Modes of particular wavelengths have developed by recombination. \(\Rightarrow\) Standard ruler at recombination

Its present angular size depends mostly on spatial curvature (and to less extent on \(\rho_\Lambda\))

That’s how we know with good precision that the space is Euclidean

- **Heights of peaks** very sensitive to baryon number density.

If not for baryons, effects of \(\delta T\) and gravitational potential at recombination would cancel each other.

That’s how bayon-to-photon ratio is measured by CMB observations
Scalar tilt and tensor amplitude

Power spectrum again: define

\[ \mathcal{P}(k) = \frac{k^3 P(k)}{2\pi^2} \]

Meaning:

\[ \langle \delta^2(x) \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}(k) \]

Parametrize

\[ \mathcal{P}(k) = A_s \cdot (k/k_*)^{n_s-1} \]

\(n_s - 1) = \text{tilt. Flat, Harrison–Zeldovich spectrum: } n_s = 1, \text{ all scales contribute equally.} \]

Predictions of inflation: (i) \(n_s\) slightly different from 1.
   (ii) Often sizeable tensor modes

Tensor modes would enhance low \(l\) region
Present situation

$r = \frac{A_T}{A_s}$

Not conclusive yet, but watch out Planck!
Further opportunity for observing tensor modes

CMB POLARIZATION

- CMB is polarized, because photons of different polarizations scatter off electrons differently.
- Scalar and tensor modes lead to different types of polarization (so called E- and B-modes, respectively).

Most promising way to search for tensor modes = gravity waves

- Planck, dedicated baloon experiments.
Dark energy

- Determines the expansion rate at late times $\Rightarrow$ Relation between distance and redshift.

- Measure redshifts ("easy") and distances by using standard candles, objects whose absolute luminosity is known.

Supernovae 1a

- Other, independent (but less precise) measurements of $\rho_\Lambda$: cluster abundance at various $z$, integrated Sachs–Wolfe effect, etc.
Distance-redshift for different models

\[
\Omega_M = 0.24, \ \Omega_\Lambda = 0.76
\]

\[
\Omega_M = 0.24, \ \Omega_{\text{curv}} = 0.76
\]

\[
\Omega_M = 1, \ \Omega_\Lambda = 0
\]
First SNe data

\[ \Omega_M = 0.28, \Omega_\Lambda = 0.72 \]

\[ \Omega_M = 0.20, \Omega_\Lambda = 0.00 \]

\[ \Omega_M = 1.00, \Omega_\Lambda = 0.00 \]
Newer SNe data
Cluster abundance

\( \Omega_M = 0.25, \ \Omega_\Lambda = 0.75, \ h = 0.72 \)

\( \Omega_M = 0.25, \ \Omega_\Lambda = 0, \ h = 0.72 \)

\( \Omega_\Lambda = 0.75 \)

\( \Omega_\Lambda = 0, \ \text{curvature domination} \)
Who is dark energy?

- Vacuum = cosmological constant
  By Lornetz-invariance

  \[ T_{\mu\nu}^{\text{vac}} = \text{const} \cdot \eta_{\mu\nu} \]

  \text{const} = \rho_\Lambda, \text{ fundamental constant of Nature.}
  \rho_\Lambda = (2 \cdot 10^{-3} \text{ eV})^4: \text{ ridiculously small.}
  No such scales in fundamental physics.
  Problem for any interpretation of dark energy

- Definition of energy density and pressure:

  \[ T_{\mu\nu} = (\rho, p, p, p) \]

  Hence, for vacuum \( p = -\rho \).

- Parametrize: \( p_{DE} = w \rho_{DE} \implies w_{\text{vac}} = -1 \)
Options

- **Quintessence**, “usual” field (modulo energy scale)
  \[ w > -1 \]

- **Phantom**: must have instabilities
  \[ w < -1 \]

\( w \) determines evolution of dark energy density:

\[
\frac{d\rho_\Lambda}{dt} = -3 \frac{\dot{a}}{a} (p_\Lambda + \rho_\Lambda)
\]

\[
\frac{\dot{\rho}_\Lambda}{\rho_\Lambda} = -3(w + 1) \frac{\dot{a}}{a}
\]
Present situation

\[ w' = \frac{dw}{dz} \]

\[ w_0 = w_{today} \]
What are we going to learn?

- Properties of primordial perturbations
  - Is there tilt in scalar spectrum?
    - Predicted by most models of inflation and many of its alternatives
  - Are there primordial gravity waves?
    - Predicted by many models of inflation but not by its alternatives
  - Are there non-Gaussianities?
    - Not in simple models of inflation, but in some more contrived inflationary models
  - Are there isocurvature scalar perturbations?
    - Not if dark matter and baryon asymmetry are generated at hot stage
- Is dark energy density constant in time?
- Is dark matter cold or warm?
- What are neutrino masses?
- Something unexpected
Effects of massive neutrinos

- Present number density of each type of neutrinos
  \[ n_{\nu_1} = n_{\nu_2} = n_{\nu_3} = 112 \frac{1}{\text{cm}^3} \]

- If neutrinos are heavier than 0.1 eV, they are degenerate in mass, \( m_{\nu_1} \approx m_{\nu_2} \approx m_{\nu_3} = m_{\nu} \)

- Relatively heavy neutrinos would make large portion of dark matter. Example: for \( m_{\nu} = 0.3 \) eV
  \[ \rho_{\nu} = 3 \cdot 0.3 \text{ eV} \cdot 112 \frac{1}{\text{cm}^3} = 100 \frac{\text{eV}}{\text{cm}^3} \approx 0.1 \rho_{DM} \]

  (Recall that
  \( \rho_{DM} \approx 0.2 \cdot \rho_c = 0.2 \cdot 5 \cdot 10^{-6} \text{ GeV} \cdot \text{cm}^{-3} = 1 \text{ keV} \cdot \text{cm}^{-3} \).)

- Neutrino is hot component of dark matter. Tends to wash out structures up to large size.
Power spectrum with neutrinos

\[ P(k) \propto (\text{Mpc}/h)^3 \]

\[ k \propto (\text{h/Mpc}) \]

- \( f_\nu = 0 \)
- \( f_\nu = 0.1 \)

\[ f_\nu = \frac{\Omega_\nu}{\Omega_{DM}}. \]
Observational data

\[ P_g(k) \left[ (h^{-1} \text{Mpc})^3 \right] \]

\[ k \left[ \text{h/Mpc} \right] \]

\[ m_\nu = 0.09 \text{ eV} \]

\[ m_\nu = 1 \text{ eV} \]

\[ m_\nu = 0.5 \text{ eV} \]
Current cosmological limit

\[ \sum_i m_{\nu_i} < 0.6 \text{ eV} \implies m_\nu < 0.2 \text{ cm} \]

NB: Stronger, but less compelling limits available.

Prospective: measure neutrino masses by cosmological methods