Spin-1/2 particle in external electro-magnetic field

Dirac equation

\[ \hbar = c = 1 \]

\[ (i \gamma^\mu \frac{\partial}{\partial x^\mu} - e \gamma^\mu A_\mu - m) \Psi(x) = 0 \]

\[ A_\mu = (\phi, \vec{A}_i) \]  
external em field

4x4 \(\gamma\)-matrices

\[ \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & 0 \\ 0 & i \sigma^i \end{pmatrix} \]
Spin-$1/2$ particle in external electro-magnetic field

Dirac equation

\[ (i \gamma^\mu \frac{\partial}{\partial x^\mu} - e \gamma^\mu A_\mu - m) \Psi(x) = 0 \]

\[ A_\mu = (\Phi, \vec{A}_i) \]

non relativistic limit

\[ \vec{p}_i \ll m \]

Rewriting

\[ i \partial_t \Psi = \left[ i \gamma^0 \gamma^k \mathbf{\nabla}_k + i \gamma^0 m + e \Phi \right] \Psi \]
Non relativistic limit

\[ E = m + \Delta E, \quad \Delta E \ll m \]

\[
\psi = \exp\left\{ -i mt \right\} \left( \begin{array}{c}
\varphi \\
\chi
\end{array} \right)
\]

\[
i \partial_t \chi = \vec{\theta} \cdot \vec{\nabla} \left( \begin{array}{c}
\chi \\
\varphi
\end{array} \right) + e\Phi(\chi) - 2m(\chi)
\]

small component

\[ \chi = \frac{\vec{\theta} \cdot \vec{\nabla}}{2m} \varphi \]

large component

\[ i \partial_t \varphi = \left( \frac{(\vec{\theta} \cdot \vec{\nabla})^2}{2m} + e\Phi \right) \varphi \]
\[ i \partial_t \psi = \left( \frac{(\vec{E} \cdot \vec{A})^2}{2m} + e \Phi \right) \psi \]

\[(\vec{E} \cdot \vec{A})(\vec{E} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i \vec{E} \cdot \varepsilon_{ikl} \vec{A}_k \vec{B}_l \Rightarrow \vec{H}_i = -\varepsilon_{ikl} \vec{A}_k \vec{A}_l = \text{rot} \vec{A} \]

\[ i \partial_t \psi = \left[ \frac{\vec{\nabla} \cdot \vec{\nabla}}{2m} - \frac{e}{2m} \vec{E} \cdot \vec{H} + e \Phi \right] \psi \]

- Kinetic energy \( \sim p^2/2m \)
- Spin interaction
- Electromagnetic potential energy

Interaction of spin with magnetic field

\[-\frac{e}{2m} \vec{E} \cdot \vec{H} = -\mu \cdot g \vec{S} \cdot \vec{H} \]

Spin operator

\[ \vec{S} = \frac{\vec{J}_0}{g}, \quad g = 2 \]

Magnetic moment

\[ \mu = \frac{e}{2m} (1 + \kappa) \]
Electron as elementary particle

anom. magnetic moment $e^-$
experiment:

$$K_e = 0.00166...$$

anom. magnetic moment $e^-$
thought $QED$ correction

magnetic moment

$$\mu = \frac{e}{2m} (1 + \kappa)$$

$$m_e = 0.51 \text{ MeV}$$

$$\sim \frac{e^2}{4\pi} \approx \frac{1}{137} \quad \text{small!}$$

Electron is elementary particle!
Proton as elementary particle

anom. magnetic moment $p$

experiment:

$K_p = 0.00$???

anom. magnetic moment $p$, theory

QED correction

magnetic moment

$$\mu = \frac{e}{2m} \left( 1 + \kappa \right)$$

$m_p = 938 \text{MeV}$

$\frac{m_e}{m_p} = 1/1840$

small!

theory 1933 experiment

Max Born Otto Stern

Victor Weisskopf reports $^3$ that Stern visited Göttingen for a seminar at that time to talk about the planned experiment. Having explained his apparatus, he asked the auditory for a prediction for the magnetic moment of the proton, and everyone of them - from Max Born to Victor Weisskopf - wrote down his prediction and signed it, one magneton with the mass of the proton, of course, as predicted by Dirac's equation.
Proton as elementary particle

anom. magnetic moment $p$

experiment:

$K_p = 0.00???$

anom. magnetic moment $p$ theory from QED correction

magnetic moment

$\mu = \frac{e}{2m} \left(1 + \kappa\right)$

$m_p = 938\text{MeV}$

theory 1933 experiment

W. Pauli  Otto Stern

Emilio Segrè: The great Pauli, who was also at Hamburg then, tried to talk Stern out of performing the experiment, because it would be a waste of time and effort, and the result was already known.
But proton... is not such elementary particle

anom. magnetic moment $p$

experiment:

O. Stern 1933: $k_p = 1.5 \pm 10\% \ (1.79284...)$

$\Rightarrow$ proton is not elementary like electron, it can have the inner structure (charge and magnetic distribution)

theory of anom. magnetic moment $k_p$ and $k_n = -1.913042...$

W. Pauli

$$\gamma^\mu \left( i \partial_\mu - e_p A_\mu \right) + \frac{k_p e}{4 M} G_{\mu \nu} F^{\mu \nu} - M \Psi_p = 0$$

Pion cloud 1950$^{th}$

$$g_{\pi N} \approx 13!$$

$k_p = 0.035g^2(1+0.87g^2+...)$

$k_n = -0.261g^2(1+0.06g^2+...)$

but $k_p/k_n \approx -1/7$

later QCD, qualitatively ...
Elastic electron-proton scattering: basic facts

\[ k = (E, 0, 0, E) \]
\[ k' = (E', 0, E' \sin \theta, E' \cos \theta) \]
\[ q^2 = (k - k')^2 = -4EE' \sin^2 \theta/2 \]

\[ Z e g(\vec{x}), \quad \int d\vec{x} g(\vec{x}) = 1 \]

\[ x \rightarrow (x) \quad \tau = -i \int d^4x A_\mu(x) j^\mu(x) \]

\[ j^\mu(x) = -e \bar{u}(k') \gamma^\mu u(k) e^{-i(k-k')x} \]

\[ A^\mu(x) = (\phi(\vec{x}), \vec{\sigma}) \quad \text{static source} \]
Elastic electron–proton scattering: basic facts

\[ \kappa = (E, 0, 0, E) \]
\[ \kappa' = (E', 0, E' \sin \theta, E' \cos \theta) \]
\[ q^2 = (\kappa - \kappa')^2 = -4EE' \sin^2 \theta / 2 \]

Target: static, spinless charge distribution

\[ Ze \sigma (\vec{x}) , \int d\vec{x} \sigma (\vec{x}) = 1 \]

\[ T = -i \int d^4x A_\mu (x) j^\mu (x) \]

\[ T = i e \bar{u}(k') \gamma^0 u(k) 2\pi \delta (E - E') \]
\[ \times \int d\vec{x} e^{-i (\vec{q} \cdot \vec{x})} \Phi (\vec{x}) \]
Elastic electron-proton scattering: basic facts

\[ k = (E, 0, 0, E) \]
\[ k' = (E', 0, E'\sin\Theta, E'\cos\Theta) \]
\[ q^2 = (k-k')^2 = -4EE'\sin^2\Theta/2 \]

Target: static, spinless charge distribution

\[ Ze g(\vec{x}) \]
\[ \int d\vec{x} g(\vec{x}) = 1 \]

\[ T = ie \bar{u}(k') \gamma^0 u(k) 2\pi \delta(E-E') \]
\[ \times \int d\vec{x} e^{-i(\vec{q} \cdot \vec{x})} F(\vec{q}^2) \]

FF is just the Fourier transform of the charge distribution

\[ F(\vec{q}^2) \overset{def}{=} \int d^3\vec{x} e^{i(\vec{q} \cdot \vec{x})} g(\vec{x}) \]
Elastic electron-proton scattering: basic facts

\[ k = (E, 0, 0, E) \]

\[ k' = (E', 0, E'\sin\theta, E'\cos\theta) \]

\[ q^2 = (k - k')^2 = -4EE'\sin^2\theta/2 \]

Target: static, spinless charge distribution

Amplitude

\[ T = i \bar{u}(k') \sigma U(k) \frac{Ze}{q^2} 2\pi \delta(E - E') F(q^2) \]

Cross section

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} |F(q^2)|^2 \]

\[ \left( \frac{d\sigma}{d\Omega} \right)_{Mott} = \frac{(Ze)^2 \cos^2\theta/2}{4E^2 \sin^2\theta/2} \]

\[ \delta(\pi') = \delta(\pi') \]
Elastic e-p scattering: ff and charge radius

\[ k = (E, 0, 0, E) \]
\[ k' = (E', 0, E'\sin \Theta, E'\cos \Theta) \]
\[ q^2 = (k - k')^2 = -4EE'\sin^2 \Theta/2 \]

\[ F(q^2) = \int d^3 \hat{x} e^{i(\vec{q} \cdot \vec{x})} \rho(r) \sim \int d^3 \hat{x} \left[ 1 + i(\vec{q} \cdot \vec{x}) + \frac{i^2}{2} (\vec{q} \cdot \vec{x})^2 + \ldots \right] \rho(r) \]

\[ \approx 1 - \frac{1}{6} \frac{q^2}{\langle r^2 \rangle} \]

Mean square radius of the charge cloud:

\[ \langle r^2 \rangle \]

Scattering on a target:

Small \( q^2 \) \( \iff \) small \( \Theta \), \( E \) fixed

\[ \frac{q^2}{\langle r^2 \rangle} \]
Elastic $e-p$ scattering: $ff$ and charge radius

\[
\begin{align*}
\kappa &= (E, 0, 0, E) \\
\kappa' &= (E', 0, E' \sin \Theta, E' \cos \Theta) \\
q^2 &= (\kappa - \kappa')^2 = -2EE' \sin^2 \Theta/2 \\
F(q^2) &= \int d^3 \hat{x} e^{i(\hat{q} \cdot \hat{x})} \rho(r) \approx \int d^3 \hat{x} \left[ 1 + i(\hat{q} \cdot \hat{x}) + \frac{i^2}{2} (\hat{q} \cdot \hat{x})^2 + \ldots \right] \rho(r) \\
&\sim 1 - \frac{1}{6} q^2 \langle r^2 \rangle
\end{align*}
\]

mean square radius of the charge cloud

Let $\rho(\vec{x}) = \rho(r)$, $r = 1 \times 1$

small $q^2 \Leftrightarrow$ small $\Theta$, $E$ fixed

scattering on the target constituents

\[ F(q^2) \]
small $q^2$ \quad large $q^2$
Elastic e-p scattering: ff and charge radius

$\kappa = (E, 0, 0, E)$

$\kappa' = (E', 0, E'\sin \Theta, E'\cos \Theta)$

$q^2 = (\kappa - \kappa')^2 = -4EE'\sin^2 \Theta/2$

$F(q^2) = \int d^3\vec{x} e^{i(q\cdot\vec{x})} \rho(r) \sim \int d^3\vec{x} \left[ 1 + i(q\cdot\vec{x}) + \frac{q^2}{2}(q\cdot\vec{x})^2 + \ldots \right] \rho(r)$

$\sim 1 - \frac{1}{6} q^2 \langle r^2 \rangle$

mean square radius of the charge cloud

Let $\rho(\vec{x}) = \rho(r)$, $r = |\vec{x}|$

small $q^2 \iff$ small $\Theta$, $E$ fixed

If $\rho(r) \sim e^{-r\lambda}$ \quad $\Rightarrow$ \quad $F(q^2) = \frac{1}{(1 + \frac{q^2}{\lambda^2})^2}$
Elastic $e$-$p$ scattering: proton target

$$\kappa = \left( E, 0, 0, E \right) \quad \mathbf{p} = (M, \vec{0})$$

$$k' = \left( E', 0, E' \sin \Theta, E' \cos \Theta \right)$$

$$q^2 = (k-k')^2 = -4EE' \sin^2 \Theta / 2$$

$$T = -i \int d^4x \left\{ -\frac{1}{q^2} \right\} J_\mu(x)$$

Electron vertex

$$J^\mu_e = -e \bar{u}(k') \gamma^\mu u(k) e^{i(k'-k)xc}$$
Elastic e-p scattering: proton target

\[ \kappa = (E, 0, 0, E) \quad p = (M, \vec{0}) \]
\[ k' = (E', 0, E'\sin\Theta, E'\cos\Theta) \]
\[ q^2 = (k-k')^2 = -4EE'\sin^2\Theta^2 \]

\[ T = -i \int d^4x \, j_e^\mu(x) \left\{ -\frac{1}{q^2} \right\} J_\mu(x) \]

proton

\[ J_\mu = e \, \overline{N}(p') \left[ \ldots \right] N(p) e^{i(p'-p)x} \]

\[ \left[ \ldots \right] = \gamma_\mu F_1(q) + \frac{k_p}{2M} i\sigma^{\mu\nu} q_\nu F_2(q) \]

Dirac ff \hspace{1cm} Pauli ff \hspace{1cm} effective int
Elastic $e-p$ scattering: proton target

\[ \kappa = (E, 0, 0, E) \quad p = (M, 0) \]
\[ \kappa' = (E', 0, E'\sin\theta, E'\cos\theta) \]
\[ q^2 = (k-k')^2 = -4EE'\sin^2\theta/2 \]

\[ T = -i \int d^4x \, j_e^\mu(x) \left\{ -\frac{i}{q^2} \right\} J_\mu(x) \]
\[ J_\mu = e \bar{N}(p') \left[ \ldots \right] N(p) e^{i(p'-p)\cdot x} \]

\[ \left[ \ldots \right] = \delta_\mu F_1(q) + \frac{k_p}{2M} \gamma_\mu q^\nu F_2(q) \]

normalization \[ F_1(0) = F_2(0) = 1 \]
\[ J^\mu_p = \frac{e}{2M} (1 + k_p) \]
Elastic $e-p$ scattering: proton form factors

Perform the non-relativistic limit in order to obtain physical interpretation (scattering on ext. em field $A_\mu$)

$$A_\mu = (\Phi, \vec{0}) \quad J_0 \simeq e \left\{ F_1(q) + \frac{q^2}{4M^2} \kappa_p F_2(q) \right\} 2M \psi^+ \psi$$

charge distrib. in proton

$$A_\mu = (\sigma, \vec{A}_k) \quad \vec{H}_i = -i \epsilon_{ijk} q_j \vec{A}_k$$

magnet. distrib. in proton

$$\tau = \frac{-q^2}{4M^2}$$

$$\vec{A}_k \vec{J}_k = 2M \psi^+ \left\{ -\frac{e}{2M} \vec{\sigma} \cdot \vec{H} \right\} \psi \left\{ F_1(q) + \kappa_p F_2(q) \right\}$$

$$G_E(q) = F_1(q) - \tau \kappa_p F_2(q) \quad G_M(q) = F_1(q) + \kappa_p F_2(q)$$

electric ff \quad magnetic ff

normalization $$G_E(0) = 1, \quad G_M(0) = 1 + \kappa_p$$
Elastic e-p scattering: cross section

\[ p_m = (H, \vec{0}) \]
\[ k_m = (E, \theta, \phi, E) \]
\[ -q^2 = Q^2 = 4EE'\sin^2\frac{\theta}{2} \]

\[ \tau = \frac{-q^2}{4M^2} \]

Convenient variables

\[ \xi = \left(1 + 2(1 + \tau)\tan^2\frac{\theta}{2}\right)^{-1}, \quad 0 \leq \xi \leq 1 \]

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{ns} \left\{ \tau \ G_M^2 + \xi \ G_E^2 \right\} \]

\[ \left( \frac{d\sigma}{d\Omega} \right)_{ns} = \frac{a^2}{4E^2} \frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} \frac{E'}{E} \]

Non-structure cross-sec

\[ \sigma_R = \tau \ G_M^2 + \xi \ G_E^2 \]

Reduced cross-sec
Elastic $e-p$ scattering: Rosenbluth plot

How to obtain ff’s from the unpolarized cross section?

Use linear dependence of the reduced cross section

$$\sigma_R = \tau G_M^2 + \varepsilon G_E^2$$

linear fit
The Beginnings

Fig. 26. Typical angular distribution for elastic scattering of 400-Mev electrons against protons. The solid line is a theoretical curve for a proton of finite extent. The model providing the theoretical curve is an exponential with rms radii $= 0.80 \times 10^{-13}$ cm.

R. Hofstadter, Rev. Mod. Phys. 56 (1956) 214

ed-elastic
Finite size + nuclear structure

Fig. 31. Introduction of a finite proton core allows the experimental data to be fitted with conventional form factors (McIntyre).

proton FF's

\[ Q^2 \leq 10 \text{ GeV}^2 \]

\[ \frac{\mu_p}{G_M} \approx 1 \]

fit: \[ \langle r^2 \rangle_E^\text{fit} = (0.89 \text{ fm})^2 \]

\[ G_E \approx \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} \]

\[ \Lambda^2 = 0.71 \text{ GeV}^2 \]

\[ \langle r^2 \rangle_E \approx (0.81 \text{ fm})^2 \]

**FF at large-\(Q^2\)**

\[ QCD: \quad F_1(Q) \sim \frac{1}{Q^4} \]

\[ F_2(Q) \sim \frac{1}{Q^6} \]

cross section at large-\(Q^2\)

\[ \frac{d\sigma}{dQ^2} \sim \frac{1}{Q^4} \quad F_1^2(Q^2) \sim \frac{1}{Q^{12}} \]

proton FF’s

\[ Q^2 \leq 10 \text{ GeV}^2 \]

\[
\frac{\mu_p G_E}{G_M} \approx 1
\]

fit: \[
\langle r^2 \rangle_E^{fit} = (0.89 \text{ fm})^2
\]

\[
G_E \approx \frac{1}{\left(1 + Q^2/\Lambda^2\right)^2}, \quad \Lambda^2 = 0.71 \text{ GeV}^2
\]

\[
\langle r^2 \rangle_E \approx (0.81 \text{ fm})^2
\]

Proton FF's

$Q^2 \leq 10 \text{ GeV}^2$

\[
\frac{\mu_p G_E}{G_M} \sim 1
\]

Figure 2.16: The world data on the ratio of electric to magnetic form factors $\mu_p G_E/G_M$ by Rosenbluth separation.

Figure 2.17: The world data on the ratio of electric to magnetic form factors $\mu_p G_E/G_M$ by recoil polarization. The dashed line is the recoil polarization published fit.

proton FF’s

$Q^2 \leq 10$ GeV$^2$

$\frac{\mu_p G_E}{G_M} \sim 1$
Elastic $e^{-}p$ scattering: polarization experiments

\[ \bar{e} + p \rightarrow e + \bar{p} \]

\[ P_t = -\sqrt{\frac{2\varepsilon(1 - \varepsilon)}{\tau}} \frac{G_E G_M}{\tau \sigma_{\text{red}}} \]
\[ P_l = \sqrt{1 - \varepsilon^2} \frac{G_M^2}{\tau \sigma_{\text{red}}} \]

The ff ratio can be measured directly

\[ \frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1 + \varepsilon)}} \frac{G_E}{G_M} \]

JLab experiments

unpolarized

- Hall C, PR C70 (2005)

\[ Q^2 = 0.4 - 5.5 \text{ GeV}^2 \]
\[ \varepsilon = 0.1 - 0.97 \]

recoil polarization

- Hall A, PR C64 (2001)
- Hall A, PR C71 (2005)

\[ Q^2 = 0.5 - 5.6 \text{ GeV}^2 \]
\[ \varepsilon \approx 0.85 \]
Discrepancy between Rosenbluth and polarization data

Rosenbluth technique (global analysis)

\[
\frac{\mu_p G_E}{G_M} \approx 1
\]

Polarization data fit:

\[
\frac{\mu_p G_E}{G_M} = 1 - 0.13(Q^2 - 0.04)
\]

\[
Q^2 = 0.5 - 5.6 \text{ GeV}^2
\]
Discrepancy between Rosenbluth and polarization data

JLab Hall A   E01-001

Precision Rosenbluth measurements

\[ Q^2 = 2.64, 3.20, 4.10 \text{ GeV}^2 \quad \varepsilon = 0.1 - 0.9 \]

Solid line – fit to E01-001 ‘Super-Rosenbluth’

Dashed line – taken from polarization transfer ratio

- \( G_E/G_M \) are in agreement with SLAC experiments

The discrepancy between the Rosenbluth separations and recoil polarization results is clearly established
Speculation: missing radiative corrections

Radiative correction at electron side:
well understood and taken care of

Box diagrams involve photons of all wavelength
long wavelength (soft photon) is included in radiative correction (IR divergence is cancelled with electron proton bremsstrahlung interference)

Soft bremsstrahlung involves long-wavelength photons
compositeness of the nucleon only enters through on-shell $f_f$

Maximon, Tjon PRC62, 2000
Elastic $e-p$ scattering: Two Photon Exchange

TPE amplitude

$$A_{ep} = \frac{e^2}{Q^2} \bar{l}' \gamma_\mu l \bar{N}' \left( \tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{P^\mu}{M^2} \gamma.K \right) N$$

$$P = \frac{1}{2} p + p' \quad K = \frac{1}{2} (k + k')$$

$$\tilde{G}_M = G_M + \delta \tilde{G}_M \quad \tilde{F}_2 = F_2 + \delta \tilde{F}_2$$

Reduced cross section with TPE correction

$$\sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M R \left( \delta \tilde{G}_M + \varepsilon \frac{\nu}{M^2} \tilde{F}_3 \right) + 2 \frac{\varepsilon}{\tau} G_E R \left( \delta \tilde{G}_E + \frac{\nu}{M^2} \tilde{F}_3 \right) + \mathcal{O}(e^4)$$

Born app

Re part of TPE
TPE at large-$Q$: QCD factorization approach

Fit of unpolarized data with the TPE exchange contribution

### Data

- **recoil polarization**
  - Hall A, PR C64 (2001)
  - Hall A, PR C71 (2005)
  - Hall C, GEp-III, prelim.

### Theory

- **COZ**
- **BLW**

### Table

<table>
<thead>
<tr>
<th>$f_N$ ($10^{-3}$ GeV$^2$)</th>
<th>$r_-$</th>
<th>$r_+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0 ± 0.5</td>
<td>4.0 ± 1.5</td>
<td>1.1 ± 0.3</td>
</tr>
<tr>
<td>5.0 ± 0.5</td>
<td>1.37</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Polarized observables with TPE: test of $\varepsilon$-dependence

$$P_l = \sqrt{1 - \varepsilon^2} \frac{1}{\sigma_R} \left\{ G_M^2 + 2 G_M \mathcal{R} \left( \frac{\varepsilon}{1 + \varepsilon M^2} \tilde{F}_3 \right) + \mathcal{O}(\varepsilon^4) \right\}$$

$$P_t = -\sqrt{\frac{2\varepsilon(1 - \varepsilon)}{\tau}} \frac{1}{\sigma_R} \left\{ G_E G_M + G_E \mathcal{R} \left( \delta \tilde{G}_M \right) + G_M \mathcal{R} \left( \delta \tilde{G}_E + \frac{\nu}{M^2} \tilde{F}_3 \right) + \mathcal{O}(\varepsilon^4) \right\}$$

TPE corrections
Golden mode: $e^+p$ vs. $e^-p$ elastic scattering

$$\frac{\sigma_{e^+p}}{\sigma_{e^-p}} \sim 1 + 4A_{1\gamma} \text{Re}A_{2\gamma}$$

direct test of real part of $2\gamma$ amplitude

Existing data: SLAC $Q^2=0.01-5\text{GeV}^2$ large errors!

Future experiments:
- JLab Class PR-07-005 $Q^2=0.5-2.5$ GeV$^2$ $\varepsilon=0.1-.9$
- VEPP-3 Novosibirsk $Q^2=1.6$ GeV$^2$ $\varepsilon=0.44, .92$
- Olympus@Desy $Q^2=0.8-4.5$ GeV$^2$ $\varepsilon=0.4-.9$

QCD estimate

![Graph showing $\sigma_{e^+/e^-}$ ratio with BLW and COZ models and data points from various experiments.](image-url)
In conclusion ...

- New experimental developments allow considerably improve our knowledge about nucleon electromagnetic elastic form factors
- Tremendous progress during last decade
- Higher precision, low systematic uncertainties through polarization experiments
- Worldwide activity at its peak

Progress in past decade
- High-$Q^2$ surprise in $G_{Ep}/G_{Mp}$, strong impact on theoretical picture. Evidence for two-photon exchange effect
- Evidence for structure beyond $G_{Dipole}$ at low $Q^2$

Many new experiments underway or proposed
- JLab Hall C E-05-017 unpol $d\sigma/d\Omega$ being analyzed, $Q^2=0.9-6.6\text{GeV}^2$
- JLab Hall A PR-07-108 unpol $d\sigma/d\Omega$ $Q^2=7-17.5\text{GeV}^2$, stat. err. < 1%
- JLab Hall A/C PR-07-109/09-001 ratio $G_{E}/G_{M}$ for proton $Q^2=6-14.8\text{GeV}^2$
- JLab Hall A PR-09-16 ratio $G_{E}/G_{M}$ for neutron $Q^2=5-10.2\text{GeV}^2$
- JLab Hall C PR-04-019 $P_{li}(\epsilon)$ at $Q^2=3\text{GeV}^2$ and $\epsilon=.13, .44, .70, .81$
- JLab Hall A PR-05-15, Target Normal SA, sensitive to Im-part of TPE (nucleon target) $Q^2=1, 2.3\text{GeV}^2$