QUARK MODELS FOR HEAVY HADRONS

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• The constituent quark model (CQM) is one of the oldest phenomenological approaches describing hadrons as composite states of quarks

• Large variety of quark models ranging from simple nonrelativistic models describing only some specific properties of hadrons (e.g. masses . . .) to complicated relativistic ones attempting to give universal description of properties of various hadrons (light and heavy)

$\Longrightarrow$ It is not possible to cover all of them in this talk

Thus only main common features of the modern potential models for the description of hadrons containing heavy quarks will be presented

PLAN

1. Introduction

   — Main assumptions/approximations of CQM
   — Heavy mesons
   — Heavy baryons
   — Decays of heavy hadrons

2. Relativistic quark model

3. Selected CQM predictions

   — B_c meson
   — Heavy baryons

4. Summary

1. INTRODUCTION

– Main assumptions of CQM:

- Hadrons consist of valence coloured quarks:

  mesons - $q\bar{q}$

  baryons - qqq

  all other states (tetraquarks, pentaquarks . . .) are called "exotic"

- Sea quarks and gluons, complicated structure of QCD vacuum (condensates . . .) etc. $\Longrightarrow$ constituent quark masses and confinement

- Interaction between quarks in hadrons can be described by the potential which is usually assumed to be flavour independent

- Hadrons are considered to be quasi-stable

– Heavy mesons

- Bound state equation - typically Schrödinger-like:

$$[T + V]\Psi = E\Psi$$

T - energy of free quarks
V - potential energy
Ψ - bound state wave function
E - eigenvalue

A. Kinematical structure

- Nonrelativistic models (Schrödinger equation)

$$T = \frac{\mathbf{p}^2}{2\mu}; \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}; \qquad E = M - (m_1 + m_2)$$

M - meson mass
$m_{1,2}$ - quark masses

- Relativistic models

a) Spinless Salpeter equation

$$T = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2}; \qquad E = M$$

b) Quasipotential equation (rationalisation of square roots)

$$T = \frac{\mathbf{p}^2}{2\mu_R}; \qquad E = \frac{b^2(M)}{2\mu_R}$$

relativistic reduced mass:

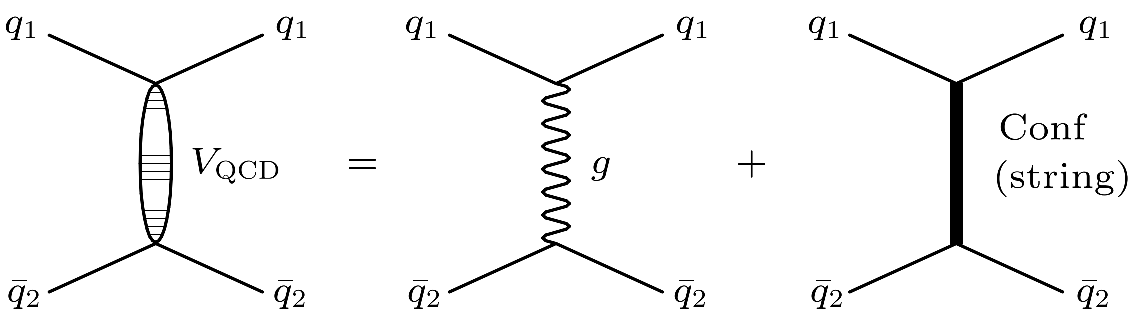
$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3} \xrightarrow{\text{weak binding}} \frac{m_1 m_2}{m_1 + m_2}$$

on-mass-shell relative momentum squared:

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

B. Dynamics

- Quark-antiquark interaction potential



- Static potential

a) One gluon exchange potential (Coulomb-like) reduces to

$$V_{\rm Coul}(r) = -\frac{4}{3}\frac{\alpha_s}{r}$$

with the running QCD coupling constant (one-loop)

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2n_f)\ln(\mu^2/\Lambda^2)}$$

n_f - number of flavours with masses below μ

$\Lambda \sim \Lambda_{\rm QCD} \sim 200$ MeV

b) Confining interaction (string?)

$$V_{\text{conf}}(r) = Ar + B$$

with string tension $A \sim 0.18$ GeV^2

Such form of static potential is supported by lattice QCD calculations.

All phenomenologically successful static potentials coincide in the distance region 0.2 fm-1.5 fm

Static (nonrelativistic) potential (Cornell)

$$V_{\text{static}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar + B$$

- Relativistic contributions to the potential

b quark	$\langle v^2/c^2 \rangle \sim 0.1$
c quark	$\langle v^2/c^2 \rangle \sim 0.3$

- Spin-dependent terms

Lorentz structure:

a) One gluon exchange potential (OGEP) $\longrightarrow$ Lorentz vector

OGEP is constructed using $q_1\bar{q}_2$ scattering amplitude (as in QED)

$$\mathcal{M} = [\bar{u}_1(p_1')\gamma^\mu u_1(p_1)][\bar{u}_2(p_2')\gamma^\nu u_2(p_2)]D_{\mu\nu}(\mathbf{k})$$

$D_{\mu\nu}(\mathbf{k})$ - gluon propagator
$u(p)$ - Dirac spinor

b) Confining potential $\longrightarrow$ Lorentz scalar, vector or mixture of scalar and vector

For the confining potential

$$\mathcal{M} = \bar{u}_1(p_1')\bar{u}_2(p_2')\mathcal{V}_{\rm conf}(\mathbf{k})u_1(p_1)u_2(p_2)$$

where

$$\mathcal{V}_{\rm conf}(\mathbf{k}) = V^S_{\rm conf}(\mathbf{k}) + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu}$$

$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark

$$V^S_{\rm conf}(\mathbf{k}) = \varepsilon V_{\rm conf}(\mathbf{k})$$

$$V^V_{\rm conf}(\mathbf{k}) = (1-\varepsilon)V_{\rm conf}(\mathbf{k})$$

ε - mixing coefficient of scalar and vector confining potentials

Spin-dependent potential up to v^2/c^2 order

$$V_{\mathrm{spin-dep}}(r) = a\,\mathbf{LS} + b\left[\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right] + c\,\mathbf{S}_1\mathbf{S}_2 + d\,\mathbf{L}(\mathbf{S}_1 - \mathbf{S}_2)$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

spin-orbit term

$$a = \frac{1}{4}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\left(\frac{4}{3}\frac{\alpha_s}{r^3} - \frac{A}{r}\right) + \frac{1}{m_1 m_2}\frac{4}{3}\frac{\alpha_s}{r^3} + \left(\frac{1}{m_1} + \frac{1}{m_2}\right)^2 (1+\kappa)(1-\varepsilon)\frac{A}{r}$$

tensor term

$$b = \frac{1}{3m_1 m_2}\left(\frac{4}{3}\frac{\alpha_s}{r^3} + (1+\kappa)^2(1-\varepsilon)\frac{A}{r}\right)$$

spin-spin term

$$c = \frac{4}{3m_1 m_2}\left(\frac{8\pi\alpha_s}{3}\delta^3(r) + (1+\kappa)^2(1-\varepsilon)\frac{A}{r}\right)$$

Charmonium $1P$ levels $(n_r\,{}^{2S+1}L_J)$

Center of gravity of ${}^3P_J$ states:

$$\bar{M}(\chi_{cJ}) = \frac{5M(\chi_{c2}) + 3M(\chi_{c1}) + M(\chi_{c0})}{9}$$

$(h_c = {}^1P_1)$

$$M(h_c) - \bar{M}(\chi_{cJ}) \approx -\langle c \rangle = -\frac{4(1+\kappa)^2(1-\varepsilon)}{3m_1m_2}\left\langle \frac{A}{r} \right\rangle$$

Experiment:

$$M(h_c) - \bar{M}(\chi_{cJ}) = -0.05 \pm 0.19 \pm 0.16 \text{ MeV}$$

$\Longrightarrow (1+\kappa)^2(1-\varepsilon) = 0$

a) $\left.\begin{matrix} (1-\varepsilon) = 0 \\ \varepsilon = 1 \end{matrix}\right\} \Longrightarrow$ scalar confinement

b) $\left.\begin{matrix} (1+\kappa) = 0 \\ \kappa = -1 \end{matrix}\right\} \Longrightarrow$ mixture of scalar and vector confining potentials (our choice!)

vanishing long-range chromomagnetic interaction !

(In agreement with the flux tube model and recent lattice QCD calculations)

– Heavy baryons

3-body problem (more complicated)

- Nonrelativistic

$$T = \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m_i}$$

- Static potential

$$V = -\sum_{i<j} \frac{2}{3} \frac{\alpha_s}{r_{ij}} + V_{\rm conf}$$

- Confining potential

a) Δ-type (pair interactions between quarks)

$$V_{\rm conf}^{\Delta} = \sum_{i<j} \frac{1}{2} A r_{ij}$$

b) Y-type

$$V_{\rm conf}^{Y} = A\ L_{\rm min}$$

$L_{\rm min}$ - the minimal length corresponding to the Y-shaped string configuration

- Spin-dependent terms:
  - Spin-spin interaction

$$V_{ij}^{\rm hyp} = \frac{16\pi\alpha_s}{3m_i m_j}\delta(r_{ij})\,\mathbf{S}_i\mathbf{S}_j$$

  - Spin-orbit $(\mathbf{LS})$ and tensor terms are often neglected

- In most models main parameters (quark masses, parameters of quark interaction potential . . .) have different values in meson and baryon sectors

- Main methods of the approximate solution of the bound state equation:
  - variational approach
  - hyperradial approximation in hyperspherical formalism
  - approximate numerical solution of Faddeev type equation

- Quark-diquark picture:

  - Baryons with one heavy quark $\longrightarrow$ heavy-quark–light-diquark picture

  - Baryons with two heavy quarks (doubly heavy) $\longrightarrow$ light-quark–heavy-diquark picture

Three-body calculation $\longrightarrow$ two-step two-body calculations

First step: calculation of diquark properties (masses, wave functions, diquark-gluon form factors)

Second step: calculation of masses and wave functions of baryons as the bound states of the diquark and quark

Diquark is a composite system with total spin $S = 0, 1$:

- diquark is not point-like: Its interaction with gluons is smeared by the form factor expressed through the overlap integral of diquark wave functions

Pauli principle for ground state diquarks:

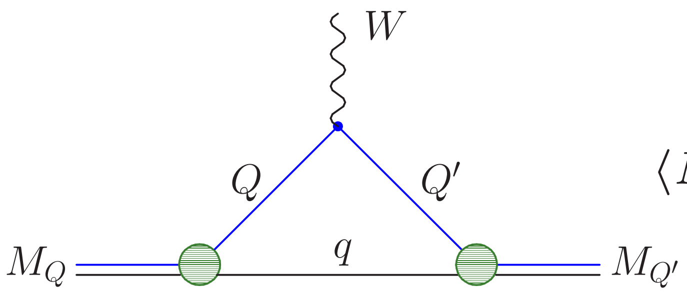
- (qq') diquark can have $S = 0, 1$ (scalar $[q, q']$, axial vector $\{q, q'\}$)
- (qq) diquarks can have only $S = 1$ (axial vector $\{q, q\}$)

– Decays of heavy hadrons

• Decay widths are significantly more sensitive to the details of the quark interaction potential and relativistic corrections to the matrix elements than mass spectra

• Decay amplitudes are calculated as convolution of quark diagrams with the wave functions of initial and final hadrons

Semileptonic decays of heavy mesons



$$\langle M_{Q'}|J_\mu(0)|M_Q\rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{M_{Q'}\mathbf{P}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{M_Q\,\mathbf{Q}}(\mathbf{q})$$

• Usually decay form factors are calculated at some kinematical point (minimal or maximal recoil of the final meson) and than extrapolated to the whole kinimatical range using some ansatz (pole, Gaussian etc.). Only in some relativistic quark models form factors are explicitly calculated for all values of momentum transfer

• Important: Weak decay matrix elements should satisfy all constrains of heavy quark symmetry (not automatically fulfilled in models)

• For relativistic calculations it is necessary to account for the relativistic transformations of hadron wave functions from the rest to moving reference frame

2. RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks (diquarks)
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

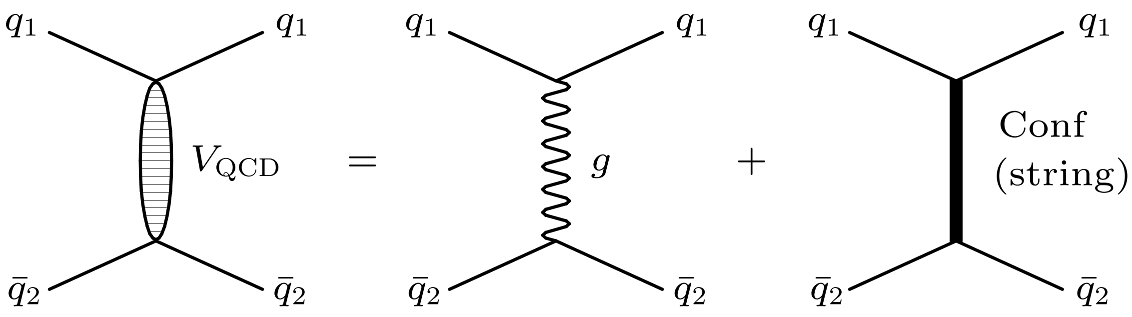
$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Heavy mesons

- $q\bar{q}$ **quasipotential**

(Constructed with the help of off-mass-shell scattering amplitude projected onto positive-energy states)



$$V(\mathbf{p}', \mathbf{p}; M) = \bar{u}_1(p')\bar{u}_2(-p')\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + V_{\mathrm{conf}}^{V}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + V_{\mathrm{conf}}^{S}(\mathbf{k})\right\}u_1(p)u_2(-p)$$

$\mathbf{k} = \mathbf{p}' - \mathbf{p}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_{\mu}(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_{\mu}(\mathbf{k}) = \gamma_{\mu} + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^{\nu},$$

κ - anomalous chromomagnetic moment of quark,

Model parameters (9)

Parameters A, B, κ, ε and constituent quark masses are fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ${}^3P_J$ states and HQET

$(1 + \kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction !

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.169$ GeV

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

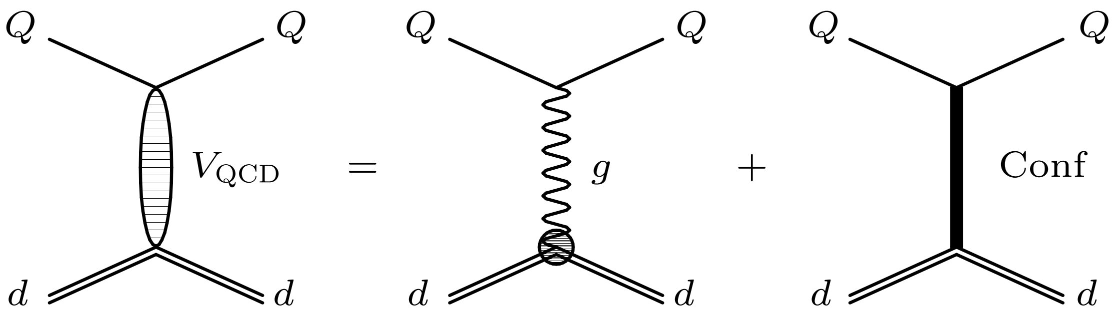
$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

- Heavy baryons in quark-diquark picture

(qq)-interaction:

$$V_{qq} = \frac{1}{2} V_{q\bar{q}}$$

(dQ)-interaction: $d = (qq')$



3. SELECTED CQM PREDICTIONS

– B_c meson

Table 1: B_c mass spectrum (in MeV).

State $n\,^{2S+1}L_J$	Our	Eichten, Quigg	Gershtein et al.	Fulcher	Godfrey	Experiment CDF	Experiment D0
$1\,^1S_0$	6270	6264	6253	6286	6271	$6274.1(3.2)(2.6)$	6300(14)(5)
$1\,^3S_1$	6332	6337	6317	6341	6338		
$1\,^3P_0$	6699	6700	6683	6701	6706		
$1\,P_1$	6734	6730	6717	6737	6741		
$1\,P_1^{'}$	6749	6736	6729	6760	6750		
$1\,^3P_2$	6762	6747	6743	6772	6768		
$2\,^1S_0$	6835	6856	6867	6882	6855		
$2\,^3S_1$	6881	6899	6902	6914	6887		
$1\,^3D_1$	7072	7012	7008	7019	7028		
$1\,D_2$	7077	7009	7001	7028	7041		
$1\,D_2^{'}$	7079	7012	7016	7028	7036		
$1\,^3D_3$	7081	7005	7007	7032	7045		
$2\,^3P_0$	7091	7108	7088		7122		
$2\,P_1$	7126	7135	7113		7145		
$2\,P_1^{'}$	7145	7142	7124		7150		
$2\,^3P_2$	7156	7153	7134		7164		
$3\,^1S_0$	7193	7244			7250		
$3\,^3S_1$	7235	7280			7272		

Semileptonic decays of B_c mesons

- B_c is the only meson consisting from two heavy quarks that decays only by weak interactions
- decays of both heavy quarks give compatible contributions to the total decay rate:

c quark decays $\sim 70\%$
b quark decays $\sim 20\%$
weak annihilation $\sim 10\%$

Table 2: Semileptonic decay rates Γ (in 10^{-15} GeV) of B_c mesons.

Decay	our	Ivanov et al.	Kiselev et al.	El-Hady et al.	Chang, Chen	Colangelo, De Fazio	Anisimov et al.	Nobes, Woloshyn	Lu et al.	Liu, Chao
b quark decay										
$B_c \to \eta_c e\nu$	5.9	14	11	11.1	14.2	2.1(6.9)	8.6	6.8	4.3	8.31
$B_c \to \eta_c' e\nu$	0.46		0.60		0.73	0.3				0.605
$B_c \to J/\psi e\nu$	17.7	33	28	30.2	34.4	21.6(48.3)	17.5	19.4	16.8	20.3
$B_c \to \psi' e\nu$	0.44		1.94		1.45	1.7				0.186
$B_c \to D e\nu$	0.019	0.26	0.059	0.049	0.094	0.005(0.03)			0.001	0.0853
$B_c \to D^* e\nu$	0.11	0.49	0.27	0.192	0.269	0.12(0.5)			0.06	0.204
c quark decay										
$B_c \to B_s e\nu$	12	29	59	14.3	26.6	11.1(12.9)	15	12.3	11.75	26.8
$B_c \to B_s^* e\nu$	25	37	65	50.4	44.0	33.5(37.0)	34	19.0	32.56	34.6
$B_c \to B e\nu$	0.6	2.1	4.9	1.14	2.30	0.9(1.0)			0.59	1.90
$B_c \to B^* e\nu$	1.7	2.3	8.5	3.53	3.32	2.8(3.2)			2.44	2.34

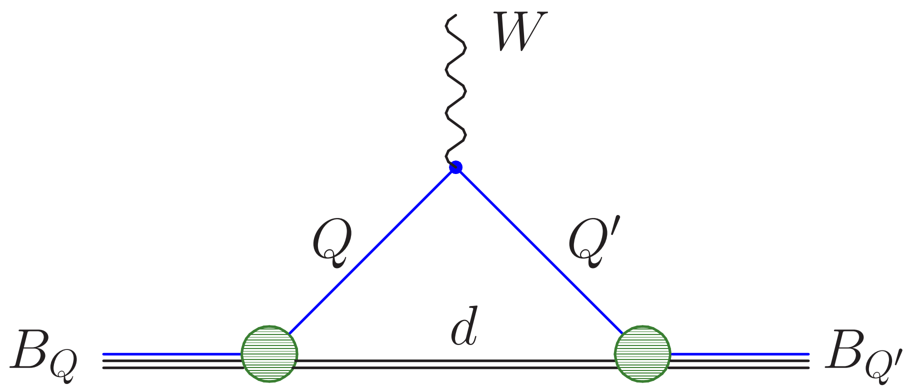
– Heavy baryons

Table 3: Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory						Experiment
		our (2005)	Capstick Isgur	Roberts Pervin	Karliner et al	Jenkins	Narodetskii et al.	PDG
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2268				2286.46(14)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2455			2454	2453.76(18)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2519			2522	2518.0(5)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2466			2460	2471.0(4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2594		2580.8(2.1)		2578.0(2.9)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2649				2646.1(1.2)
Ω_c	$0(\frac{1}{2}^+)$	2698		2718				2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2776		2760.5(4.9)		2768.3(3.0)†
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5612				5620.2(1.6)
Σ_b	$1(\frac{1}{2}^+)$	5805	5795	5833	5814	5824.2(9.0)	5808	5807.5(2.5)‡
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5858	5836	5840.0(8.8)	5833	5829.0(2.3)‡
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5806	5795(5)	5805.7(8.1)	5791	5792.9(3.0)*
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5970	5930(5)	5950.9(8.5)		
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980	5959(4)	5966.1(8.3)		
Ω_b	$0(\frac{1}{2}^+)$	6065		6081	6052(6)	6068.7(11.1)		
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6102	6083(6)	6083.2(11.0)		

† BaBar 2006; ‡ CDF 2006 (Σ_b^+); * CDF 2007

Semileptonic decays of heavy baryons



$$Br^{\text{theor}}(\Lambda_b \to \Lambda_c l\nu) = 6.9\% \qquad (|V_{cb}| = 0.041,\ \tau_{\Lambda_b} = 1.23 \times 10^{-12}\text{s})$$

Experiment

$$Br^{\text{exp}}(\Lambda_b \to \Lambda_c l\nu) = \begin{cases} \left(5.0^{+1.1+1.6}_{-0.8-1.2}\right)\% & \text{DELPHI} \\ \left(8.1 \pm 1.2^{+1.1}_{-1.6} \pm 4.3\right)\% & \text{CDF} \end{cases}$$

$$Br^{\text{exp}}(\Lambda_b \to \Lambda_c l\nu + \text{anything}) = (9.1 \pm 2.1)\%. \qquad \text{PDG}$$

Masses of doubly heavy baryons

Table 4: Masses of the ground-state doubly heavy baryons (in GeV)

Baryon	Quark content	J^P	our	Gershtein et al.	Martynenko	Albertus et al.	Körner et al.	Narodetskii et al.	Roberts Pervin
Ξ_{cc}	$\{cc\}q$	$1/2^+$	3.620	3.478	3.510	3.612	3.61	3.69	3.676
Ξ^*_{cc}	$\{cc\}q$	$3/2^+$	3.727	3.61	3.548	3.706	3.68		3.753
Ω_{cc}	$\{cc\}s$	$1/2^+$	3.778	3.59	3.719	3.702	3.71	3.86	3.815
Ω^*_{cc}	$\{cc\}s$	$3/2^+$	3.872	3.69	3.746	3.783	3.76		3.876
Ξ_{bb}	$\{bb\}q$	$1/2^+$	10.202	10.093	10.130	10.197		10.16	10.340
Ξ^*_{bb}	$\{bb\}q$	$3/2^+$	10.237	10.133	10.144	10.236			10.367
Ω_{bb}	$\{bb\}s$	$1/2^+$	10.359	10.18	10.422	10.260		10.34	10.454
Ω^*_{bb}	$\{bb\}s$	$3/2^+$	10.389	10.20	10.432	10.297			10.486
Ξ_{cb}	$\{cb\}q$	$1/2^+$	6.933	6.82	6.792	6.919		6.96	7.020
Ξ'_{cb}	$[cb]q$	$1/2^+$	6.963	6.85	6.825	6.948			7.047
Ξ^*_{cb}	$\{cb\}q$	$3/2^+$	6.980	6.90	6.827	6.986			7.074
Ω_{cb}	$\{cb\}s$	$1/2^+$	7.088	6.91	6.999	6.986		7.13	7.136
Ω'_{cb}	$[cb]s$	$1/2^+$	7.116	6.93	7.022	7.009			7.165
Ω^*_{cb}	$\{cb\}s$	$3/2^+$	7.130	6.99	7.024	7.046			7.187

Semileptonic decays of doubly heavy baryons

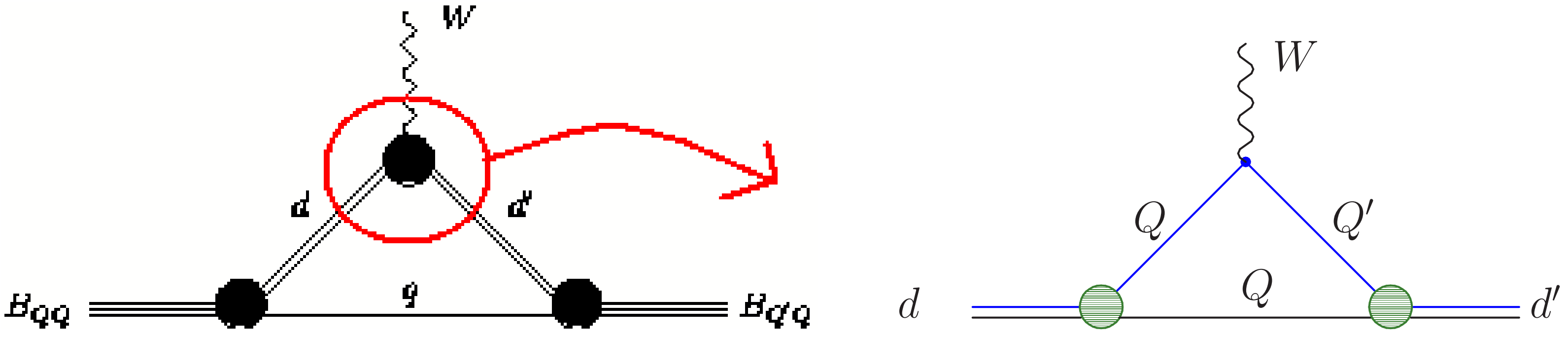


Table 5: Semileptonic decay widths of doubly heavy baryons Ξ_{bb} and Ξ_{bc} ($\times 10^{-14}$ GeV).

Decay	our	Guo et al.	Lozano	Onischenko et al.	Ivanov et al.	Albertus et al.
$\Xi_{bb} \to \Xi'_{bc}$	0.82	4.28				1.06
$\Xi_{bb} \to \Xi_{bc}$	1.63	28.5		8.99		1.92
$\Xi_{bb} \to \Xi^*_{bc}$	0.53	27.2		2.70		
$\Xi^*_{bb} \to \Xi'_{bc}$	0.82	8.57				
$\Xi^*_{bb} \to \Xi_{bc}$	0.28	52.0				
$\Xi^*_{bb} \to \Xi^*_{bc}$	1.92	12.9				
$\Xi'_{bc} \to \Xi_{cc}$	0.88	7.76				1.36
$\Xi'_{bc} \to \Xi^*_{cc}$	1.70	28.8				
$\Xi_{bc} \to \Xi_{cc}$	2.30	8.93	4.0	8.87	0.8	2.57
$\Xi_{bc} \to \Xi^*_{cc}$	0.72	14.1	1.2	2.66		
$\Xi^*_{bc} \to \Xi_{cc}$	0.38	27.5				
$\Xi^*_{bc} \to \Xi^*_{cc}$	2.69	17.2				

4. SUMMARY

- Although CQM is based on assumptions and approximations that cannot be directly derived from QCD it provides a useful tool for studying the properties of hadrons
- The uncertainties of calculations can be reliably estimated only in the framework of the model itself. The accuracy of the quark model approximations is unknown
- At present for many properties of hadrons (such as excited hadron states, multiquark states . . .) it is the only tool that can give numerical predictions
- Many predictions of quark models were confirmed by experiment

“My conclusion is that if you want to know the mass of a particle and if you have little time (in years!) and little money you should forget all your prejudices and use potential models. This is, in fact, even true to a large extent for systems containing light quarks, which is still more mysterious.”

André Martin (CERN) CERN-TH/96-318

BACKUP SLIDES

Rationalisation of quasipotential equation

$$\epsilon_i = \sqrt{\mathbf{p}^2 + m_i^2}$$

$$(M - \epsilon_1 - \epsilon_2)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q})$$

Multiplying both sides by

$$(M + \epsilon_1 + \epsilon_2)[M^2 - (\epsilon_1 - \epsilon_2)^2]$$

we get

$$([M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2] - 4M^2\mathbf{p}^2)\Psi_M(\mathbf{p}) = 4M^2(b^2(M) - \mathbf{p}^2)\Psi_M(\mathbf{p})$$

$$= (M + \epsilon_1 + \epsilon_2)[M^2 - (\epsilon_1 - \epsilon_2)^2] \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q})$$

with

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

Dividing by $8M^2\mu_R$ with

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

we get

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = I(M; \mathbf{p}) \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q})$$

with

$$I(M; \mathbf{p}) = \frac{(M + \epsilon_1 + \epsilon_2)[M^2 - (\epsilon_1 - \epsilon_2)^2]}{8ME_1E_2} \xrightarrow{\text{on mass shell}} 1$$

- **Heavy baryons in quark-diquark picture**

(qq)-interaction: $$V_{qq} = \frac{1}{2} V_{q\bar{q}}$$

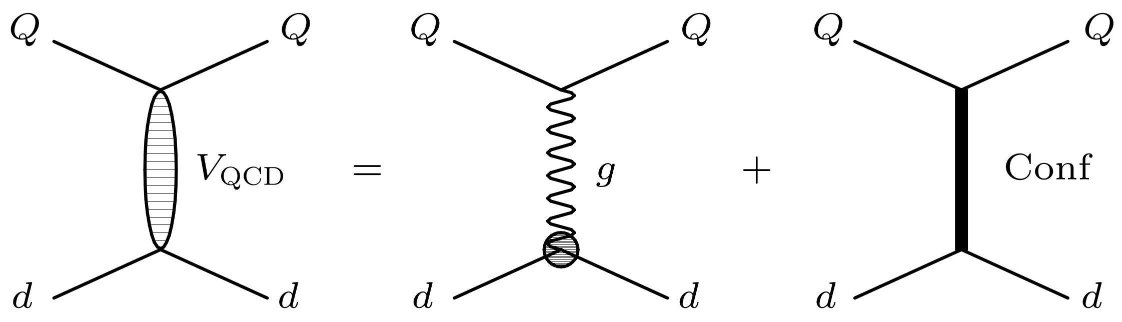
$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + \frac{1}{2}V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + \frac{1}{2}V^S_{\rm conf}(\mathbf{k})$$

(dQ)-interaction: $$d = (qq')$$

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q)$$
$$+\psi^*_d(P)\bar{u}_Q(p)J_{d;\mu}\Gamma^\mu_Q V^V_{\rm conf}(\mathbf{k})u_Q(q)\psi_d(Q) + \psi^*_d(P)\bar{u}_Q(p)V^S_{\rm conf}(\mathbf{k})u_Q(q)\psi_d(Q)$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark } (\mu_{\mathrm{d}} = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

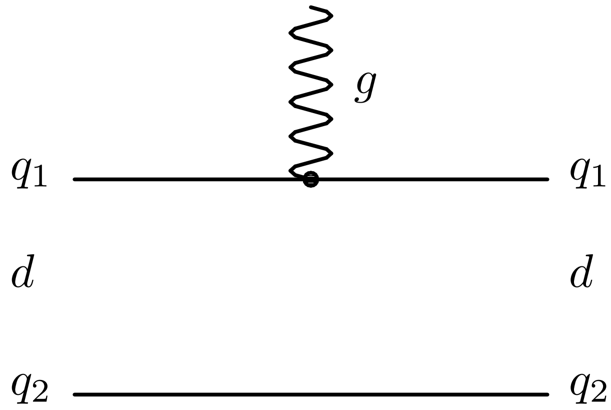


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

Table 6: Masses of the excited Λ_Q ($Q = c, b$) baryons (in MeV) (scalar diquark)

$I(J^P)$	Qd state	$Q = c$		$Q = b$		
		M(our)	$M^{\rm exp}$ PDG	M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ CDF
$0(\frac{1}{2}^+)$	$1S$	2297	2286.46(14)	5622	5624(9)	5619.7(2.4)
$0(\frac{1}{2}^-)$	$1P$	2598	2595.4(6)	5930		
$0(\frac{3}{2}^-)$	$1P$	2628	2628.1(6)	5947		
$0(\frac{1}{2}^+)$	$2S$	2772	2766.6(2.4)?	6086		
$0(\frac{3}{2}^+)$	$1D$	2874		6189		
$0(\frac{5}{2}^+)$	$1D$	2883	2882.5(2.2)	6197		
$0(\frac{1}{2}^-)$	$2P$	3017		6328		
$0(\frac{3}{2}^-)$	$2P$	3034		6337		

Table 7: Masses of the excited Σ_Q ($Q = c, b$) baryons (in MeV) (axial vector diquark)

$I(J^P)$	Qd state	$Q = c$				$Q = b$	
		M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ BaBar	$M^{\rm exp}$ Belle	M(our)	$M^{\rm exp}$ CDF
$1(\frac{1}{2}^+)$	$1S$	2439	2453.76(18)			5805	5807.5(2.5)
$1(\frac{3}{2}^+)$	$1S$	2518	2518.0(5)			5834	5829.0(2.3)
$1(\frac{1}{2}^-)$	$1P$	2805				6122	
$1(\frac{1}{2}^-)$	$1P$	2795				6108	
$1(\frac{3}{2}^-)$	$1P$	2799	$2802(^{4}_{7})$			6106	
$1(\frac{3}{2}^-)$	$1P$	2761	2766.6(2.4)?			6076	
$1(\frac{5}{2}^-)$	$1P$	2790				6083	
$1(\frac{1}{2}^+)$	$2S$	2864				6202	
$1(\frac{3}{2}^+)$	$2S$	2912		2939.8(2.3)?	$2938(^{3}_{5})$?	6222	

Table 8: Masses of the excited Ξ_Q ($Q = c, b$) baryons with scalar diquark (in MeV).

$I(J^P)$	Qd state	$Q=c$			$Q=b$	
		M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ BaBar	M(our)	$M^{\rm exp}$ CDF
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2481	2471.0(4)		5812	5792.9(3.0)
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2801	2791.9(3.3)		6119	
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2820	2818.2(2.1)		6130	
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2923			6264	
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3030			6359	
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3042		3054.2(1.3)	6365	
$\frac{1}{2}(\frac{1}{2}^-)$	$2P$	3186			6492	
$\frac{1}{2}(\frac{3}{2}^-)$	$2P$	3199			6494	

Table 9: Masses of the excited Ξ_Q ($Q = c, b$) baryons with axial vector diquark (in MeV).

		$Q = c$				$Q = b$
$I(J^P)$	Qd state	M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ Belle	$M^{\rm exp}$ BaBar	M(our)
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2578	2578.0(2.9)			5937
$\frac{1}{2}(\frac{3}{2}^+)$	$1S$	2654	2646.1(1.2)			5963
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2934				6249
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2928				6238
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2931				6237
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2900				6212
$\frac{1}{2}(\frac{5}{2}^-)$	$1P$	2921				6218
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2984		2978.5(4.1)	2967.1(2.9)	6327
$\frac{1}{2}(\frac{3}{2}^+)$	$2S$	3035				6341
$\frac{1}{2}(\frac{1}{2}^+)$	$1D$	3132				6420
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3127				6410
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3131				6412
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3123			3122.9(1.3)	6403
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3087		3082.8(3.3)	3076.4(1.0)	6377
$\frac{1}{2}(\frac{7}{2}^+)$	$1D$	3136				6390

Table 10: Comparison of theoretical predictions for masses (in MeV) of charmed baryons (for $J = \frac{1}{2}, \frac{3}{2}$) with experimental data.

J^P	exp.	our	Capstick	Migura	Garcilazo	exp.	our	Capstick	Migura	Garcilazo
			Λ_c					Σ_c		
$\frac{1}{2}^+$	2286	2297	2265	2272	2292	2454	2439	2440	2459	2448
$\frac{1}{2}^+$	2766?	2772	2775	2769	2669		2864	2890	2947	2793
$\frac{3}{2}^+$		2874	2910	2848	2906	2518	2518	2495	2539	2505
$\frac{3}{2}^+$		3262	3035	3100	3061		2912	2985	3010	2825
$\frac{1}{2}^-$	2595	2598	2630	2594	2559	2802?	2795	2765	2769	2706
$\frac{1}{2}^-$		3017	2780	2853	2779	2802?	2805	2770	2817	2791
$\frac{3}{2}^-$	2628	2628	2640	2586	2559	2766?	2761	2770	2799	2706
$\frac{3}{2}^-$		3034	2840	2874	2779	2802?	2799	2805	2815	2791
			Ξ_c					Ω_c		
$\frac{1}{2}^+$	2471	2481		2469	2496	2698	2698		2688	2701
$\frac{1}{2}^+$	2578	2578		2595	2574		3065		3169	3044
$\frac{3}{2}^+$	2646	2654		2651	2633	2768	2768		2721	2759
$\frac{3}{2}^+$		3030			2951		3119			3080
$\frac{1}{2}^-$	2792	2801		2769	2749		3020			2959
$\frac{1}{2}^-$		2928			2829		3025			3029
$\frac{3}{2}^-$	2818	2820		2771	2749		2998			2959
$\frac{3}{2}^-$		2900			2829		3026			3029

Table 11: Comparison of different predictions for semileptonic decay rates Γ ($10^{10}\mathrm{s}^{-1}$) of bottom baryons.

Decay	our RQM	Singleton NRQM	Cheng NRQM	Körner NRQM	Ivanov RTQM	Ivanov BS	Cardarelli LF	Albertus NRQM	Huang sum rule
$\Lambda_b \to \Lambda_c e\nu$	5.64	5.9	5.1	5.14	5.39	6.09	5.08 ± 1.3	5.82	5.4 ± 0.4
$\Xi_b \to \Xi_c e\nu$	5.29	7.2	5.3	5.21	5.27	6.42	5.68 ± 1.5	4.98	
$\Sigma_b \to \Sigma_c e\nu$	1.44	4.3			2.23	1.65			
$\Xi_b' \to \Xi_c' e\nu$	1.34								
$\Omega_b \to \Omega_c e\nu$	1.29	5.4	2.3	1.52	1.87	1.81			
$\Sigma_b \to \Sigma_c^* e\nu$	3.23				4.56	3.75			
$\Xi_b' \to \Xi_c^* e\nu$	3.09								
$\Omega_b \to \Omega_c^* e\nu$	3.03			3.41	4.01	4.13			