The LLog ressumation for the singular part of pion GPD

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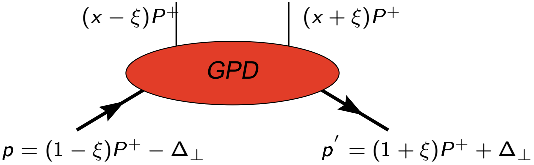
Definition of GPD



The pion GPD is defined as:

$$iT_I^{abc}H^I(x,\xi,\Delta_\perp) \;=\; \int \frac{d\lambda}{2\pi} e^{-ixP^+\lambda}\langle\pi^b(p^{'})|O^c(\lambda)|\pi^a(p)\rangle$$

$$O^c(\lambda) \;=\; \bar{q}(-\frac{\lambda n}{2})\gamma^+ q(\frac{\lambda n}{2})$$

In the forward limit $\xi\to 0$, $\Delta_\perp\to 0$ the GPD is the parton distribution function.

$$H(x,0,0) \;=\; q(x)$$

Chiral perturbation theory (ChPT)

The ChPT provides a systematic method for discussing the consequences of the global flavor symmetries of QCD at low energy. The effective Lagrangian express in terms of hadronic degrees of freedom. These are particles from the pseudoscalar octet (π, K, η) which are regarded as the Goldstone bosons of the spontaneous breaking of the chiral $SU(3)_L \times SU(3)_R$ symmetry down to $SU(3)$.
The ChPT Lagrangian contain infinite number of terms.

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^+ + \chi^+ U + U^+ \chi\right]$$

$$U(x) = \exp(i\pi^a(x)\tau^a/F_\pi) \ .$$

Effective operator

For investigation of GPD in ChPT we have to introduce the analog quark-quark operator in terms of ChPT. This analog should satisfy all properties and symmetries of initial operator. At lowest order:

$$O(\lambda) = \frac{iF_\pi^2}{4} F(\beta,\alpha) *$$

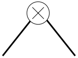
$$\mathrm{tr}\left[U\Big(\frac{\alpha+\beta}{2}\lambda n\Big)\partial_+ U^+\Big(\frac{\alpha-\beta}{2}\lambda n\Big) + U^+\Big(\frac{\alpha+\beta}{2}\lambda n\Big)\partial_+ U\Big(\frac{\alpha-\beta}{2}\lambda n\Big)\right]$$

$$\simeq \frac{iF_\pi^2}{4} F(\beta,\alpha) * \left[\pi\Big(\frac{\alpha+\beta}{2}\lambda n\Big)\partial_+ \pi\Big(\frac{\alpha-\beta}{2}\lambda n\Big) \;+\; \mathcal{O}(\pi^3)\right]$$

$F(\beta,\alpha)$ is the generating function of the tower of low-energy constants.

Tree level

Using usual rules for calculation in ChPT we build the row of Feynman diagrams, for GPD. The tree level gives GPD chiral limit at $\Delta_\perp = 0$.

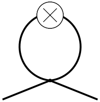


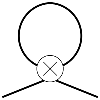
$$\mathring{H}^I(x,\xi,0) \;=\; F^I(\beta,\alpha) * \Big[\delta(x-\xi\alpha-\beta) - (1-I)\xi\delta(x-\xi(\alpha+\beta))\Big]$$

From it one can see that $F(\beta,\alpha)$ is Double Distribution function for pion GPD in chiral limit.

Next-to-leading contribution

At the next-to-leading order one has two 1-loop diagram:





$$H^{I=1}(x,\xi,\Delta) \;=\; \mathring{H}^{I=1}(x,\xi,0)\left[1-\frac{m_\pi^2\ln m_\pi^2}{(4\pi F_\pi)^2}\right]+$$

$$\frac{a_\chi}{2}\frac{\theta(|x|<\xi)}{\xi}\int_{-1}^{1}d\eta R[\eta,t]\ln(R[\eta,t])\frac{d}{d\eta}\mathring{H}\left(\frac{x}{\xi\eta},\frac{1}{\eta},0\right)$$

$$R[\eta,t]=\frac{1}{(4\pi F_\pi)^2}\left(m_\pi^2-(1-\eta^2)\frac{t}{4}\right)$$

$$a_\chi \;=\; \frac{m_\pi^2}{(4\pi F_\pi)^2}\approx 0.014$$

Singular contribution

Let's look closely previous expression:

$$H^{I=1}_{NLO}(x,\xi,\Delta) \;\sim\; a_\chi \frac{\theta(|x|<\xi)}{\xi}$$

At small $\xi < a_\chi$ this "small" correction is not small, moreover at forward limit it produce singularity.

$$H_{NLO}(x,0,0) \;=\; q_{NLO}(x) \;=\; a_\chi \ln(1/a_\chi)\delta(x)$$

Singular terms at higher orders

One can obtain that at NNLO ChPT theory gives more singular term

$$H_{NNLO}(x,\xi,\Delta) \sim a_\chi^2 \frac{\theta(|x|<\xi)}{\xi^2}\,, \quad q_{NLO}(x) \sim a_\chi^2 \ln^2(1/a_\chi)\delta'(x)$$

Investigating the structure of diagrams one can find that such singular contribution would appear in every order. At n-order term of ChPT GPD has singularity of $\theta(|x|<\xi)/\xi^n$-type (or $\delta^{(n-1)}(x)$ in forward limit).

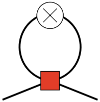
The convolution of GPD with the leading order coefficient function gives:

$$A(\xi,\Delta_\perp) = \int_{-1}^{1} \frac{H(x,\xi,\Delta_\perp)}{x-\xi}dx = A^{reg}(\xi,\Delta_\perp) + \sum_k \frac{1}{\xi^k} A_k^{sing}(\Delta_\perp)$$

One has to perform the resummation of such contributions in order to solve the problem.

Structure of singular terms

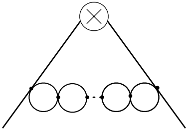
The singular term always proportional to maximal power of chiral logarithm (so called Leading Log, LLog).



Singular terms goes from special class of diagrams, which have such ($\longleftarrow$) structure. Where in red box all one-particle irreducible graphs.

Any mass appeared in numerator decrease the power of singularity.

Large-N approach



One can use large-N approach for calculation the singular contributions. Leading term is given by triangle diagram with propagation constructed from bubbles.

The answer is very simple:

$$-\frac{2}{N}\delta^{(n)}(x)\frac{\epsilon^{n+1}}{(n+1)!}\int_0^1 \beta^n q(x)\, dx$$

$$\epsilon = \frac{N}{2}a_\chi \ln(1/a\chi) \approx 0.09$$

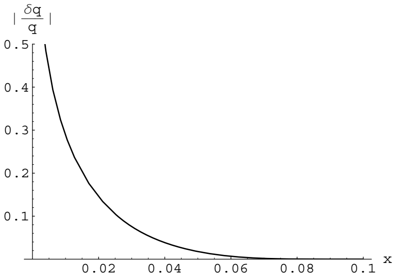
And similar for GPD.

Ressummation of singular part in large-N approach

Obtained answer for large-N approach can be exactly summed in smooth function:

$$\delta q^{l=1}_{Large-N}(x) = \sum_{n=0,2,4,..}^{\infty} -\frac{2}{N}\delta^{(n)}(x)\frac{\epsilon^{n+1}}{(n+1)!}\int_0^1 \beta^n q(x)\, dx =$$

$$-2\frac{\theta(|x|<\epsilon)}{N}\int_{|x|/\epsilon}^1 \frac{q(\beta)}{\beta}d\beta$$



LLog coefficients in renormolizable theory

Well known that in renormolizable theory using renorminvariance one can connect coefficients instead of RG-log.

For example in massless ϕ^4-theory for 4-point function the LLog coefficients a_n connects to each other by recursive relation:

$$a_n = \beta_1 a_{n-1}$$

where β_1 is one-loop β-function.

Can we obtain similar for ChPT?

Recursive relation for LLog in ChPT

In the massless ChPT the recursive equation for the LLog coefficients take a form

$$\omega_{nC} = \frac{1}{n-1}\sum_{i=1}^{n-1}\sum_{A,B}\beta(i,A;n-i,B/C)\omega_{iA}\omega_{n-i,B}$$

$$\omega_{1,0}=1\ , \quad \omega_{i,C>i} = 0$$

This allow us to calculate LLog coefficient(numerically, has not solved yet exactly).
It approved by 3-loop calculation, leading and next-to-leading order large-N calculation.

Singular part of pion GPD

For the singular part of pion GPD at the n-chiral order the answer is following:

$$\delta q_{n-\mathrm{order}}(x) \;=\; \langle x^{n-1}\rangle \delta^{(n-1)}(x)(a_\chi \ln(1/a_\chi))^{n-1}\frac{(-1)}{n!}\sum_{C=0}^{n}\omega_{nC}$$

The miracle thing is that the ω_n is falls down as $\sim \left(\frac{3}{2}\right)^n$.

Conclusion

- Presented results for large-N expansion for singular part of GPD
- Shown that all order resummation of singular terms produce smooth function
- Found the way to exact resummation of singular part
- Presented method can be used in other tasks

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