Unstable particles: from quantum mechanics to quantum field theory

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Outline

- Introduction
- How to observe the unstable particles ?
- Decay in quantum mechanics
- Quantum field theory description
- How does it work?
- Special case: fermion resonance

Main historical steps

- Discovery of radioactivity – A.H.Becquerel,1896
- α, β, γ rays – E.Rutherford, 1899
- Quantum mechanical model of α-decay – G.Gamov, 1928; R.Gurney and E.Condon, 1928
- Breit-Wigner formula – 1936
- 50 years — Hadronic and leptonic era
- Discovery of W and Z – CERN, 1983
- Discovery of Higgs boson – CERN, 2013
- (Non)-decay of proton
- Dark matter decay?

Introduction

The problem of unstable particles: understanding of physics of decay and its description has a long history starting from the discovery of a natural radioactivity by H.Becquerel in 1896. At present we know hundreds of ("elementary"or not) particles and only few of them are stable.

Time of life: from $\tau = 887\ s$ (neutron) to $\tau = 2.6 \cdot 10^{-25}\ s$ (Z^0).

Let us mention here result of search of proton decay: $\tau > 10^{33}$ years.

For short-living particles the measured value is the decay width instead of time of life

$$\Gamma = \frac{\hbar}{\tau}$$

Typical numbers

Typical weak decay (muon):

$$\tau \sim 10^{-6}\ s, \quad \Gamma \sim 10^{-16}\ \text{MeV}, \quad \Gamma/M \sim 10^{-18}$$

Typical strong decay (ρ-meson):

$$\Gamma \sim 100\ \text{MeV}, \quad \tau \sim 10^{-23} s, \quad \Gamma/M \sim 0.1$$

Typical gauge boson (Z):

$$\Gamma \sim 1\ \text{GeV}, \quad \tau \sim 10^{-25} s, \quad \Gamma/M \sim 0.01$$

Higgs boson:

$$\Gamma < 1.7\ \text{GeV}, \quad \tau > 4 \cdot 10^{-25} s, \quad \Gamma/M < 0.014$$

t-quark:

$$\Gamma \sim 1.4\ \text{GeV}, \quad \tau \sim 4 \cdot 10^{-25} s, \quad \Gamma/M \sim 0.01$$

Classical description

The classical view on radioactivity is very simple. One should suppose only **a)** radioactive nuclei have some probability to decay and **b)** this probability does not depend on the past history of individual decaying nuclei.
In other words, decays are independent and the process does not keep memory of the past. In this case we can write differential equation.
Let $N(t)$ – number of nuclei

$$dN(t) = -\frac{1}{\tau}N(t)\ dt$$

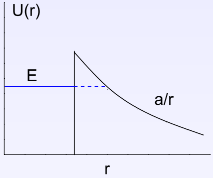
which gives the exponential law of decay

$$N(t) = e^{-t/\tau}N(0)$$

In such description we did not efforts to understand the mechanism of decay. In spite of it, the exponential law correspond very well to experimental facts.

Quantum-mechanical view

In quantum mechanics one can suggest a model of decay. A meta-stable state decays due to tunnel effect



Such a quantum-mechanical explanation was suggested firstly by G.Gamov (famous russian-american physicist) and independently by american physicists R.Gurney and E.Condon.

They discussed the α-decays of nuclei and the paper of Gamov was more quantitative: he obtained theoretically the observed relation between energy of α and the time of decay and also estimated the size of nuclei.

"Ideological"problems

It was recognized at the begining of quantum era that the existence of unstable states is in fact some challenge for quantum mechanics.

The standard quantum mechanics (based on Hilbert space axioms) is the theory of stable states and reversible time evolution. In contrast, quasistable states which can be studied in scattering or decay experiments are related with an asymmetric (iirreversable) time evolution.

The main problem: if we introduce quasistable states $|\Psi_D\rangle$ into theory, we come to states with complex eigenvalue

$$\hat{H}|\Psi_D\rangle| = (E - i\gamma)\ |\Psi_D\rangle$$

with exponential time evolution (Gamov vectors). But such vectors do not exist in the Hilbert space and such modification will lead to some contradictions.
People invented some heuristic approachs which allow to avoid main contradictions (Feynman, Gell-Mann, ...).

"Ideological"problems

But the most correct way (common opinion) is to change the main axioms of quantum mechanics

$$\text{Hilbert Space} \Rightarrow \text{Rigged Hilbert Space (Hardy Space)}$$

Few names and books:

G.Gamov, Z.Phys. 51 (1928) 204
J.von Neumann, Math. foundations of QM , Prinston, 1955.
A.Bohm, Quantum mechanics: Foundations and Applications, 2001.

George Gamov,
1904-1968

John von Neumann,
1903-1957

How to observe a long-living particle?

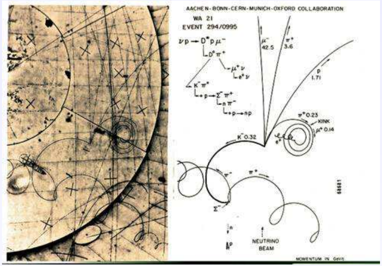


Рис. : Particles in bubble chamber with magnetic field

How to observe a short-living particle?

The method: to study distributions over invariant masses

(I) In some cases resonance is seen 'by eye'

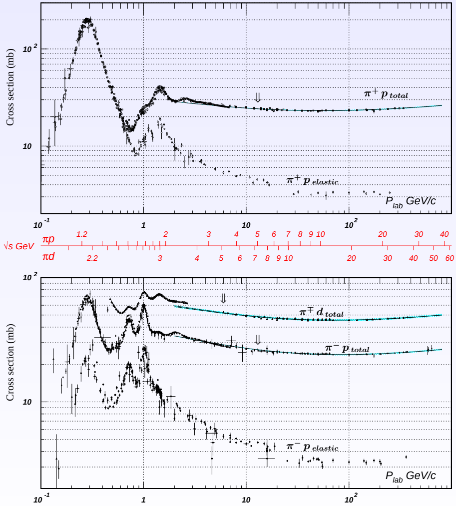
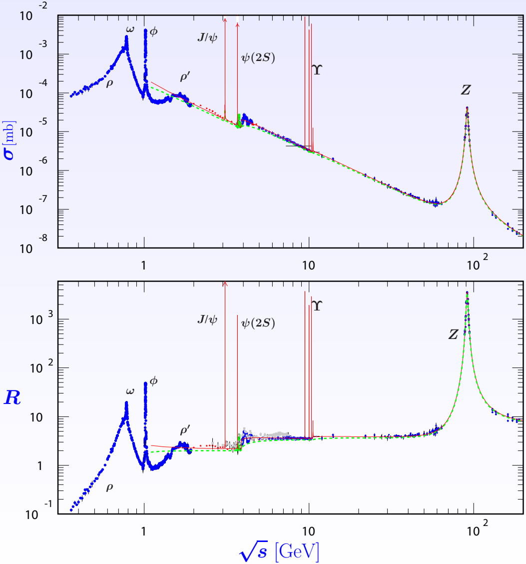


Рис. : Spin-3/2 resonance $\Delta(1232)$, clear seen in all cross-sections.

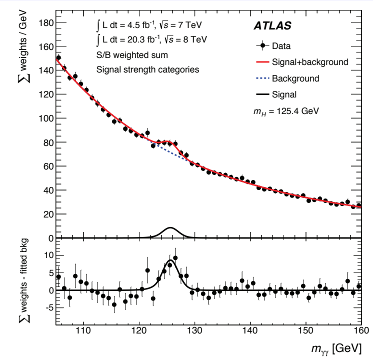
How to observe a short-living particle?

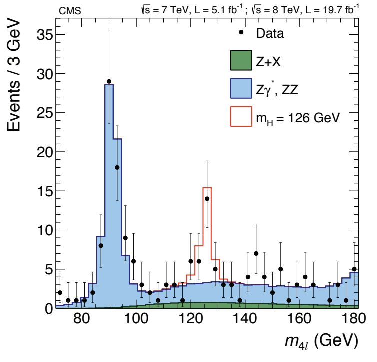
Another example: total cross-section $e^+e^- \rightarrow all$.



How to observe a short-living particle?

Higgs boson producting at LHC with huge background





How to observe a short-living particle?

(II) Partial analysis in $2 \to 2$ reactions

$$T(E,\theta) = \sum_l (2l+1) a_l(E) P_l(cos\ \theta)$$

Unitary condition for partial amplitudes

$$Im\ a_l = p|a_l|^2, \quad \Rightarrow \quad a_l = \frac{e^{2i\delta} - 1}{2ip}$$

From measured angular distributions (after not so simple procedure) it's possibly to extract the partial waves

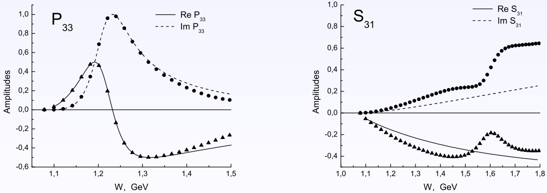


Рис. : Results of partial analysis of πN scattering with $I = 3/2$ (R.A.Arndt et. al. PR C74 (2006) 045205)

Breit-Wigner formula

Famous paper of Breit and Wigner appeared in 1936.
G.Breit and E.Wigner, Capture of Slow Neutrons, Phys. Rev. 49 (1936) 519.
Non-relativistic BW formula for propagator of unstable particle

$$\frac{k}{M - E - i\Gamma/2}$$

Relativistic variant:

$$G(p) = \frac{1}{M^2 - p^2 - iM\Gamma}$$

Gregory Breit, 1899-1981

Eugene Wigner, 1902-1995

Unstable system in the rest – QM view

Let $|\Phi_0\rangle$ – unstable state in rest at initial moment $t = 0$
Probability to survive at time t is

$$P_0(t) = |A_0(t)|^2$$

Here $A_0(t)$ is the non-decay amplitude

$$A_0(t) = \langle\Phi_0|e^{-i\hat{H}t}|\Phi_0\rangle$$

and $\hat{H}$ is the Hamiltonian of the system.

- Note, that decaying system can not be the eigenvector of $\hat{H}$. Otherwise, $e^{-i\hat{H}t} \to e^{-iE_0t}$ and $P_0(t) = 1$ — there is no decay.

But there is always a possibility to expand $|\Phi_0\rangle$ over the basis of Hamiltonian eigenvectors

$$\hat{H}|m\rangle = m|m\rangle$$

Suppose m to be a continual variable and eigenvector are normalized as

$$\langle m|m'\rangle = \delta(m - m')$$

Unstable system in the rest

Decomposition of unstable state

$$|\Phi_0\rangle = \int dm\ \rho(m)\ |m\rangle,$$

where $\rho(m) = \langle m|\Phi_0\rangle$ and

$$\int dm\ |\rho(m)|^2\ = 1$$

Here $|\rho(m)|^2$ is distribution on mass (energy in the rest frame) of unstable system, which defines a non-decay probability (Krylov,Fock)

$$A_0(t) = \int dm\ |\rho(m)|^2\ e^{-imt}.$$

If to use the Breit-Wigner distribution

$$|\rho(m)|^2_{BW} = \frac{\Gamma/2\pi}{(m-M)^2+\Gamma^2/4},$$

we obtain the standard radiative decay law

$$P_0^{BW}(t) = e^{-\Gamma t}$$

Unstable system in the rest

BUT: It is well known, that exponential law is breaking at small and large t .

- Since any physical $\hat{H}$ should have a ground state, integral has a finite low limit, $\Rightarrow$ at large t a non-decay probability has a power behavior (L.Halphin, 1958)

In order to show it, let us integrate $A_0(t)$ "by parts". It's a standard way to get an asymptotic expansion.

$$A_0(t) = \int_{m_0}^{\infty} dm \ |\rho(m)|^2 \ e^{-imt} =$$

$$= \frac{1}{it} e^{-im_0 t} \frac{d}{dm} |\rho(m)|^2 (m_0) + \frac{1}{it} \int_{m_0}^{\infty} dm \ e^{-imt} \frac{d}{dm} |\rho(m)|^2.$$

One more integration "by parts"will give $1/t^2$ term + integral.

Unstable system in the rest

$$A_0(t) = \frac{1}{it} e^{-im_0 t} \frac{d}{dm} |\rho|^2 (m_0) -$$
$$- \frac{1}{t^2} e^{-im_0 t} \frac{d^2}{dm^2} |\rho|^2 (m_0) - \frac{1}{t^2} \int_{m_0}^{\infty} dm \; e^{-imt} \frac{d^2}{dm^2} |\rho(m)|^2 .$$

So, we see a power behavior at $t \to \infty$.

- If the distribution has finite mean value

$$\bar{m} \equiv \int dm \; m \; |\rho(m)|^2 \; < \infty,$$

then derivative of non-decay probability at t=0 is equal to zero.

Unstable system in the rest

The proof is straightforward. Consider derivative of probability

$$\frac{d}{dt}P_0(t) = \frac{d}{dt}A_0(t)A_0^*(t) = \frac{dA_0}{dt}A_0^*(t) + A_0\frac{dA_0^*}{dt}$$

and put $t = 0$. From definition of amplitude $A_0(t)$ we see

$$\frac{d}{dt}A_0(t)|_{t=0} = -i\bar{m}, \quad \frac{d}{dt}A_0^*(t)|_{t=0} = i\bar{m}, \quad A_0(0) = A_0^*(0) = 1$$

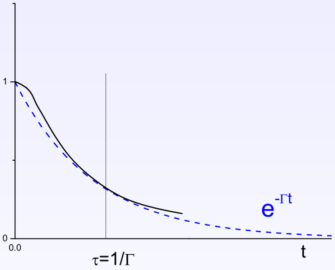
and two terms in derivative cancels at $t = 0$.
Looking again at the Breit-Wigner distribution

$$|\rho(m)|^2_{BW} = \frac{\Gamma/2\pi}{(m-M)^2 + \Gamma^2/4},$$

we see, the mean value $\bar{m}$ is indefinite. So, these arguments do not work for Breit-Wigner distribution.

Unstable system in the rest

So, the exponential law of decay should be only some approximation.



Does it have some practical sense? May be, in search of proton decay (L.Khalfin).

Quantum Zeno effect

Let us write the non-decay probability as

$$P(t) = e^{-\gamma(t)t},$$

where $\gamma(t)$ decreases at low t and tends to zero at $t \to 0$.
Such behavior of $\gamma(t)$ may be considered as consequence of so called quantum Zeno effect (Misra, Sudarshan,1977). Namely, the subsequent (quantum !) measurements and therefore subsequent collapses of the wave function of unstable state generate a slower decay rate and in limiting case can lead to a complete inhibition of decay.

Let we make N measurements with time interval Δt. Probability to survive at $T = \Delta t \cdot N$ is

$$P(T) = (P(\Delta t))^N = (e^{-\gamma(\Delta t)\Delta t})^N = e^{-\gamma(\Delta t)T}$$

Compare it with non-decay probability for a single measurement at moment T: $P(T) = exp(-\gamma(T)T)$,

Quantum Zeno effect

One can see inequality

$$e^{-\gamma(\Delta t)T} > e^{-\gamma(T)T}, \quad \text{if} \quad \gamma(\Delta t) < \gamma(T)$$

That is Zeno effect – the measurements decrease the decay probability.

Note, that in opposite case: $\gamma(\Delta t) < \gamma(T)$ the measurements lead to faster decay – anti-Zeno effect.

The quantum Zeno and anti-Zeno effects were observed in experiments with cold atoms.

Decay of moving system

We know about time dilatation in relativistic theory. So, it is natural to expect a similar dilatation for time of life

$$\tau_v = \tau_0 \cdot \gamma, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad \Gamma_v = \Gamma_0/\gamma, \quad \gamma \gg 1$$

or for non-decay probability

$$P_v(t) = P\left(\frac{t}{\gamma}\right) \qquad \textbf{(Special Relativity)}$$

This relation was checked in numerous experiments with particles.

Nevetheless, up to now one can find in literature contraversial discussions on methods to obtain similar relations and on accuracy of the Special Relativity recipe.

Decay of moving system

To illustrate the problem, let us repeat the above derivating for moving unstable system. Amplitude to survive:

$$A_v(t) = \langle \phi_v | e^{-i\hat{H}t} | \phi_v \rangle$$

Eigenvector:

$$\hat{H}|E_m(\vec{p}), \vec{p}, m\rangle = E_m(\vec{p})|E_m(\vec{p}), \vec{p}, m\rangle, \quad E_m(\vec{p}) = \sqrt{m^2 + \vec{p}^2}$$

Normaliztion:

$$\langle m' | m \rangle = \delta(m - m')$$

In the rest frame:

$$|\phi_0\rangle = \int dm \; \rho(m) \; |E_m(\vec{p}) = m, \vec{p} = 0, m\rangle$$

In the moving system

$$E_m(\vec{p}) = \gamma m, \qquad \vec{p} = \gamma m \vec{v}$$

Decay of moving system

These states are related by unitary operator

$$U_v \; |E_m(\vec{p}) = m, \vec{p} = 0, m\rangle \;\; = \;\; |E_m(\vec{p}), \vec{p}, m\rangle$$

So, vector of unstable system in the moving system

$$|\phi_v\rangle = U_v \; |\phi_0\rangle \;\; = \int dm \; \rho(m) \cdot |E_m(\vec{p}) = \gamma m, \vec{p} = \gamma m \vec{v}, m\rangle$$

After it we have the non-decay amplitude

$$A_v(t) = \int dm \; |\rho(m)|^2 \; e^{-i\gamma m t}$$

As compaired with rest frame: $t \to \gamma t$. For probability to survive we have

$$P_v(t) = P_0(\gamma t) \qquad \text{instead of expected} \qquad P_0\left(\frac{t}{\gamma}\right)$$

We have obtained a **wrong result** from "natural"arguments.

- Example is taken from Alavi, Giunty (2016)

Decay of moving system

What is the reason of wrong answer?

We used the time evolution operator $e^{-i\hat{H}t}$. But in Special Relativity one has take into accout both time and space evolution and to write down space-time dependent amplitude

$$A_v(t,\vec{x}) = \langle \phi_v | e^{-i\hat{H}t + i\vec{P}\vec{x}} | \phi_v \rangle$$

But after it we have to concretize the question, what do we observe.

About decay law in QFT

We should start from relativistic Breit-Wigner (or Breit-Wigner-like) propagator

$$G(x-y) = \int d^4p e^{-ip(x-y)}\ G(p)$$

If to use simplest BW with fixed M and Γ – all is transparent

$$G(x-y) = \int d^3p e^{i\vec{p}(\vec{x}-\vec{y})}\ \int dp^0 \frac{1}{M^2 - p_0^2 + \vec{p}^2 - i\Gamma M}$$

Now we can integrate over p_0, transforming integral into a contour one. If $t \equiv x^0 - y^0 > 0$, we can close the contour in the bottom half-plane. There exists one pole at the point

$$p_0 = +\sqrt{M^2 + \vec{p}^2 - i\Gamma M} \approx \sqrt{M^2 + \vec{p}^2} - \frac{i\Gamma M}{2\sqrt{M^2 + \vec{p}^2}} =$$

$$= \sqrt{M^2 + \vec{p}^2} - \frac{i\Gamma}{2\gamma}, \qquad \gamma = \frac{\sqrt{M^2 + \vec{p}^2}}{M}$$

About decay law in QFT

After p_0 integration

$$G(x-y) = (-2\pi i) \int d^3p e^{i\vec{p}(\vec{x}-\vec{y})} \cdot e^{-i\sqrt{M^2+\vec{p}^2}t} e^{-\Gamma\gamma t/2}$$

So, we reproduced the Special Relativity time exponent. Of course, there exist some corrections for this result

- We neglected by $(\Gamma/M)^2$ terms
- The propagator should be placed in diagram with initial and final states
- To account correctly the experimental conditions, it's better to use the wave-packet approach. It also will give some corrections to the naive exponential law.

And last, the most difficult question: what if to use the correct Breit-Wigner-like propagator (details are below), obtained by Dyson summation?

$$G(p) = \frac{1}{M^2(p^2) - p^2 - iM\Gamma(p^2)}$$

QFT and resonances

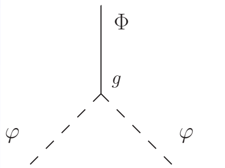
To obtain a Breit-Wigner-like formula if QFT one should account an interaction in a propagator

$$G_0(x-y) \Rightarrow G(x-y) = \frac{i\langle 0|T\{\Phi(x)\Phi(y)S\}|0\rangle}{\langle 0|S|0\rangle}$$

Toy model:

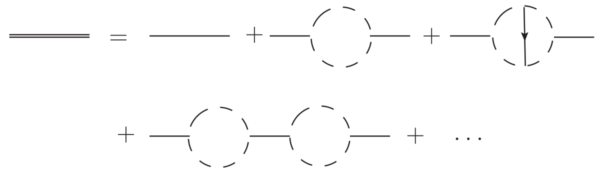
$$L_{int} = g\Phi(x)\varphi(x)\varphi(x)$$

Vertex:

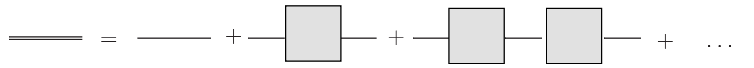


QFT and resonances

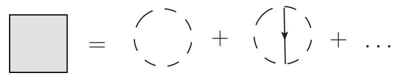
Pertubative corrections for propagator:



After recollecting:



Here box represents the sum of compact diagrams (self-energy)



QFT and resonances

Now one can recognize the geometric series

$$G = G_0 + G_0 \Sigma G_0 + G_0 \Sigma G_0 \Sigma G_0 + \dots =$$
$$= G_0 \left(1 + G_0\Sigma + (G_0\Sigma)^2 + (G_0\Sigma)^3 + \dots\right) = G_0 \frac{1}{1 - G_0\Sigma}$$

Finally after Dyson summation:

$$G(p^2) = \frac{1}{M_0^2 - p^2 - \Sigma(p^2)}$$

After renormalization:

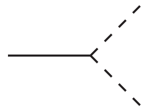
$$Re\ G^{-1} = M_0^2 - p^2 - Re\ \Sigma(p^2) = M^2 - p^2 - Re\ \Sigma^{ren}(p^2)$$

In vicinity of physical mass

$$Re\ G^{-1} = M^2 - p^2 + o(M^2 - p^2)$$

QFT and resonances

If $M > 2m$, i.e. the decay $\Phi \to \varphi\varphi$ is possible



the self-energy $\Sigma(p^2)$ has imaginary part

$$Im\ G^{-1} \equiv \Theta(M^2 - 4m^2) M\Gamma(p^2).$$

As a result, we obtain a Breit-Wigner-like expression with "running"mass and width

$$G^{ren} = \frac{1}{M^2(p^2) - p^2 - iM\Gamma(p^2)}$$

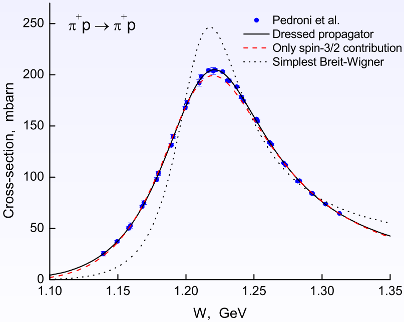
where $M^2(p^2), \Gamma(p^2)$ are slow functions.

How does it work?

There are few examples, where the form of resonance curve is measured with high accuracy.

$$\boxed{\Delta(1232),\ S = 0, J = I = 3/2}$$

$$\Delta^{++} = uuu,\ \Delta^{+} = uud,\ \Delta^{0} = udd,\ \Delta^{-} = ddd$$



How does it work?

Another example: $e^+e^- \to \rho^0(770) \to \pi^+\pi^-$

$$\boxed{\rho(770),\ J = 1,\ I = 1,\ S = 0}$$

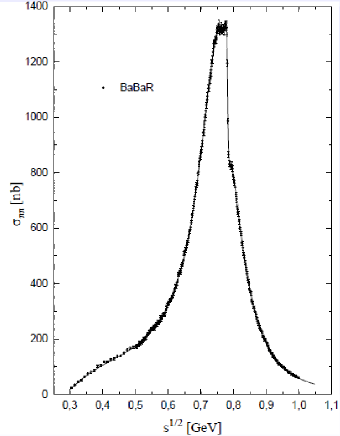
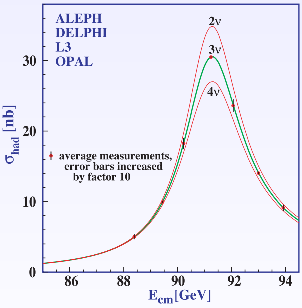


Рис. : Experiment – data from BaBar detector, curve – fit of Achasov,Kozhevnikov (2011) in "field inspired approach"

How does it work?

Neutral gauge boson

$$\boxed{Z^0,\ M = 91.2\ Gev,\ \Gamma = 2.5\ Gev}$$



Renormalization of resonance: details

After Dyson summation we have the dressed propagator

$$Re\ G^{-1} = M_0^2 - p^2 - \Sigma(p^2) = M^2 - p^2 - \Sigma^{ren}(p^2)$$

OMS scheme of renormalization says that $M_0 = M$, so we have

$$\Sigma^{ren}(p^2) = \Sigma(p^2) - \Sigma(M^2) - \Sigma'(M^2)(p^2 - M^2)$$

Generalization for the case of unstable particle

$$\Sigma^{ren}(p^2) = \Sigma(p^2) - Re\ \Sigma(M^2) - (Re\ \Sigma)'(M^2)(p^2 - M^2)$$

However, for gauge theory it was found
Pole scheme: if to continue dressed propagator into complex $s = p^2$ plane, it has a pole at second Riemann sheet

$$G(s) \sim \frac{c}{s - s_R}, \qquad s_R = (M_{Pole})^2 - iM_{Pole}\Gamma_{Pole}$$

Pole position gives another (gauge independent) parametrization of broad state.

Renormalization of resonance: details

Different definitions of mass and width:

$$(M_{BW},\ \Gamma_{BW}) \quad \Leftrightarrow \quad (M_{Pole},\ \Gamma_{Pole})$$

In principle they are related but ...

Does it lead to some difference ?

In RPP we can find two sets of parameters.

- $\Delta(1232)$:
  $M_{BW} = 1230\ to\ 1234\ Mev\ \ ,\Gamma_{BW} = 114\ to\ 120\ Mev$
  $M_{Pole} = 1209\ to\ 1211\ Mev\ \ ,\Gamma_{Pole} = 98\ to\ 102\ Mev$
- For Z boson the difference even more

$$|M_{Pole} - M_{BW}| \sim 10 \cdot \Delta M_{stat}$$

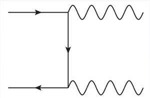
But in gauge theories there appear a new problem: violation of gauge invariance.

Problem of finite width of W Z

Simple recipe "pole $\Rightarrow$ Breit-Wigner"does not work in gauge theories (A.Sirlin, 1991). It was wound that a standard definition leads to gauge dependence of Z^0 mass in $O(\alpha^3)$ terms.
In R_ξ gauge, i.e. with gauge fixing term

$$L_{gf} = -\frac{1}{2\xi}(\partial A)^2$$

it means the appearence of ξ-dependence.
Another example: we want to observe (and describe) the process $e^+e^- \to W^+W^-$. If gauge bosons were stable

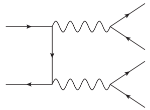


But in fact they decay and we observe the process $e^+e^- \to W^+W^- \to 4f$.

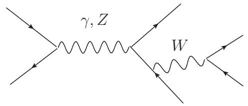
Problem of finite width of W Z

There are lot of diagrams

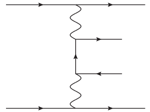
- Double-resonance (interesting for us)



- One-resonance (example)



- Additional diagrams, if there are e of ν_e (example)



- Ghost contributions in R_ξ

Problem of finite width of W Z

One can see that first class of diagrams is not gauge-invariant, so for G-I answer we should consider all diagrams $e^+e^- \to 4f$.

From theoretical point of view one should restore the gauge invariance in calculations. There are few more correct approaches for this problem

- Pole scheme (A.Sirlin, R.Stuart)
- Pinch-technics (A.Pilafsis et al)
- Backgroung-field method (A.Denner et al.)

From more practical point of view we should estimate the necessary accuracy of calculations and compaire with level of GI violation.

About fermion resonance

For bosons there exists simple parametrization (Breit-Wigner formula)

$$G(p) = \frac{1}{M^2 - p^2 - iM\Gamma},$$

or its QFT-improved variant. For fermion resonance the naive recipes do not satisfy the main requirements

$$G_0(p) = \frac{\hat{p} + m}{m^2 - p^2} \Rightarrow G(p) = \frac{\hat{p} + m}{m^2 - p^2 - im\Gamma}$$

In QFT we know the modification of inverse propagator (Dyson summation)

$$G^{-1} = \hat{p} - m - \Sigma(p), \quad \Rightarrow \quad G = \frac{1}{\hat{p} - m - \Sigma(p)}$$

How to transform it to something Breit-Wigner-like?

About fermion resonance

Trick: let us us the off-shell projection operators:

$$\Lambda^{\pm} = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \qquad (1)$$

where $W = \sqrt{p^2}$ is invariant mass or rest-frame energy.
Let us suppose, that there is no P-parity violation in the theory, so the self-energy is also decomposed in this basis

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) = \Lambda^{+}(A + WB) + \Lambda^{-}(A - WB) \equiv \Lambda^{+}\Sigma_1(W) + \Lambda^{-}\Sigma_2(W). \qquad (2)$$

Inverse propagatior:

$$S(p) = \Lambda^{+}[W - m - \Sigma_1(W)] + \Lambda^{-}[-W - m - \Sigma_2(W)]$$

Dressed propagator:

$$G(p) = \Lambda^{+}\frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^{-}\frac{1}{-W - m_0 - \Sigma_2(W)}$$

About fermion resonance

In this representation we should renormasize the scalar coefficients $G_1(W), G_2(W)$.

The positive energy pole should be compared with Breit–Wigner formula

$$G_1(W) = \frac{1}{W - m_0 - \Sigma_1(W)} \sim \frac{1}{W - m + \imath\Gamma/2}. \tag{3}$$

in vicinity of physical mass m. For negative energy pole one can use the symmetry property $G_2(W) = G_1(-W)$.

About fermion resonance

In case of parity violation:

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) + \gamma^5 C(p^2) + \hat{p}D(p^2)$$

Useful (starting) basis

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^+\gamma^5.$$

Such problem is solved by the spectral representation of inverse propagator

$$S = \lambda_1 \Pi_1 + \lambda_2 \Pi_2, \tag{4}$$

where Π_k are projectors, satisfying the eigenstate problem

$$S\Pi_k = \lambda_k \Pi_k. \tag{5}$$

Reversing it, we obtain propagator $G(p)$

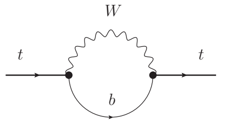
$$G = \frac{1}{\lambda_1}\Pi_1 + \frac{1}{\lambda_2}\Pi_2, \tag{6}$$

About fermion resonance

So the factor $1/\lambda_1$ is just resonance factor in theory with γ^5

$$\frac{1}{\lambda_1(W)} \sim \frac{1}{W - m + \imath\Gamma/2}. \tag{7}$$

Consider the dressing of top quark in the **Standard Model**. The main one-loop contribution to self-energy arises from Wb intermediate state



$$\Sigma(p) = \hat{p}(1 - \gamma^5)\Sigma_0(W^2). \tag{8}$$

Eigenvalue

$$\lambda_{1,2} = -m \pm W\sqrt{1 - 2\Sigma_0(W^2)}.$$

About fermion resonance

In analogy with on-mass-shell (OMS) scheme of renormalization, we should subtract the real part of self-energy at resonance point

$$\lambda_{1,2} = -m \pm W\sqrt{1 - 2\big(\Sigma_0(W^2) - Re\Sigma_0(m^2)\big)}.$$

As a result we have rather unusual resonance factor

$$\frac{1}{\lambda_1(W)} = \frac{1}{W\sqrt{1 + \imath\dfrac{\Gamma}{m}} - m}, \tag{9}$$

which only at $\Gamma/m \ll 1$ returns to standard Breit–Wigner form,

$$\frac{1}{\lambda_1(W)} \simeq \frac{1}{W - m + \imath W\dfrac{\Gamma}{2m}} \quad \text{at } \Gamma/m \ll 1.$$

Some references

L.Fonda, G.C.Ghirardi and F.Rimini. Decay theory of unstable quantum system. Rep.Prog.Phys. 41 (1978)

L.Maiani and M.Testa. Unstable system in relativistic quantum field theory. 1997

A.Bohm and Y.Sato, Relativistic resonances: Their masses, widths, lifetimes, superpositions, and causal evolution, PR D71 (2005) 085018

S.A.Alavi and C.Giunti. Which is the quantum decay law of relativistic particles? Europhys.Lett. 109 (2015) 60001

K.Urbanowski. Phys.Lett. B737 (2014) 346

F. Giacosa. Decay law and time dilatation. Acta Phys. Polon. B47 (2016) 2135

Thank you for attention!